

7 Feb 6
A R I T H M E T I C,

RATIONAL and PRACTICAL

W H E R E I N

The properties of **NUMBERS** are clearly pointed out,
the **THEORY** of the science deduced from first principles,
the methods of **OPERATION** demonstratively explained,
and the whole reduced to **PRACTICE** in a great variety of useful **RULES**.

Consisting of **THREE PARTS**, *viz.*

I. VULGAR ARITHMETIC.

II. DECIMAL ARITHMETIC.

III. PRACTICAL ARITHMETIC.

By **JOHN MAIR, A.M.**

P A R T II. 1.2 Ed

DECIMAL ARITHMETIC.

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100

Section of Division

W H E N
R E D U C T I O N

The To reduce a vulgar fraction to a decimal fraction, divide the numerator by the denominator, and annex zeros to the remainder, as often as necessary, to bring it to the desired degree of exactness. The quotient will be the decimal fraction.

A D D I T I O N

1. To add vulgar fractions, reduce them to a common denominator, and then add the numerators together. The sum of the numerators will be the numerator of the sum, and the common denominator will be the denominator of the sum.

S U B T R A C T I O N

1. To subtract vulgar fractions, reduce them to a common denominator, and then subtract the numerator of the subtrahend from the numerator of the minuend. The remainder will be the numerator of the difference, and the common denominator will be the denominator of the difference.

MUSEUM
BRITAN
NICUM

1. To multiply vulgar fractions, multiply the numerators together, and the denominators together. The product of the numerators will be the numerator of the product, and the product of the denominators will be the denominator of the product.

D I V I S I O N

1. To divide vulgar fractions, invert the divisor, and then multiply the dividend by the inverted divisor. The product will be the quotient.

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Page 3: The table ought to stand as follows:

Page 3: The table ought to stand as follows:

p. 35. r. Ex. 1. as follows.

15 s.	=	.75
7 d.	=	.02916666
$\frac{1}{2}$ d.	=	.00364583
Ans.		.78281250

p. 117. l. 21. r. 6.793

p. 119. Ex. IV. The second and third particular products should stand as follows.

6785
50866

p. 133. l. 13. the three commas ought to stand between 2 and 8, as under.

First pr. 7142,8 | 57142, | 85714 | 2, continued.

p. 150. ex. 3. point the quot thus, (.11347

ex. 4. point the first product thus, 75.0

p. 170. quest. 7. l. 3. r. Anf. .9856 +, equal to 59 min. 8 sec. - nearly.

p. 170. quest. 8. l. 2. r. Anf. 364:24579 +, equal to L. 364:4:11 nearly.

p. 174. Ex. II. l. 3. r. = $\begin{matrix} L. & s. & d. & f. \\ 2 & 5 & 6 & 14 \end{matrix}$

Ex. III. $\frac{1}{\text{ult. } r.} = 61 \ 15 \ 6 \ 21$

p. 175. Ex. V. l. 6. r. = 32 9 0 3

Ex. VI. $l. 5. r. = 86$ 4 8 0 6

Ex. VII. *l. penult. for &c. r. +*

p. 182. Ex. I. I. r. What is the product of 24 7 by, or.

P. 198. Ex. III. L4. r. = $\begin{matrix} H. & f. & g. & p. \\ 70 & 5 & 6 & 54 \end{matrix}$

ARITHMETIC,

RATIONAL and PRACTICAL

PART II.

DECIMAL ARITHMETIC; or, The Doctrine of DECIMAL FRACTIONS.

CHAP. I.

NOTATION.

A FRACTION having 10, 100, 1000, or unity with any number of ciphers annexed to it, for a denominator, is called a *decimal fraction*; such as $\frac{1}{10}$, $\frac{2}{100}$, $\frac{3}{1000}$, $\frac{8}{10000}$.

In decimal fractions, as in vulgar, the denominator shews into how many parts the unit or integer is divided, and the numerator shews how many of these parts the fraction contains. Thus, if the fraction be $\frac{9}{10}$, the unit is divided into ten equal parts, and the fraction contains nine of these parts; and consequently, if the unit or integer be a pound Sterling, the value of such a fraction is eighteen shillings.

We may conceive the denominator of a decimal fraction to be formed by dividing the unit into 10 equal parts, and each of these parts into 10 other equal parts, each of these again into 10 other equal parts, and so on, as far as necessary; and hence a decimal fraction will always be so many tenths, or so many tenths of $\frac{1}{10}$, or so many tenths of $\frac{1}{10}$ of $\frac{1}{10}$, &c.; and by reducing the

compound fraction to a simple one, we have the decimal. Thus, $\frac{8}{10}$ of $\frac{1}{10}$ of $\frac{1}{10} = \frac{8}{1000}$.

Or we may conceive the denominator of a decimal to be formed by the continual multiplication of unity into 10, as often as there are ciphers in it. Thus, $1 \times 10 = 10$, and $1 \times 10 \times 10 = 100$, and $1 \times 10 \times 10 \times 10 = 1000$, &c. And because the fractions $\frac{9}{10}$, $\frac{99}{100}$, $\frac{999}{1000}$, &c. have the highest numerators possible, it is plain, that the number of figures or places in the numerator of a decimal can never exceed the number of ciphers in the denominator.

It is usual to write down only the numerator of a decimal fraction, omitting the denominator; and when the numerator has the same number of figures or places as the denominator has ciphers, it is done by writing down the figures of the numerator, and prefixing a point, to distinguish them from a whole number. So $\frac{7}{10}$ is written thus, .7; and $\frac{25}{100}$ is written thus, .25. The point thus prefixed is called the *decimal point*.

But when the numerator has not so many figures or places as there are ciphers in the denominator, the defect is supplied by prefixing a cipher for every figure wanting, and then placing the decimal point on the left. So $\frac{3}{100}$ is written thus, .03; and $\frac{75}{10000}$ thus, .0075; and $\frac{5}{100000}$ thus, .0005.

From this manner of notation, it is easy to read a decimal, or to know its denominator, *viz* imagine 1 to stand under the decimal point, and a cypher under every decimal place. Thus, 9 is $\frac{9}{10}$, and .48 is $\frac{48}{100}$, and .05 is $\frac{5}{100}$, and .007 is $\frac{7}{1000}$, and .00036 is $\frac{36}{100000}$.

Hence it is plain, that decimals, like integers, decrease from the left to the right, and increase from the right to the left, in a decuple proportion. Thus, 5 units, or 5, by being removed one place from that of units toward the right hand, becomes .5, or $\frac{5}{10}$; and being removed one place more toward the right, it becomes .05, or $\frac{5}{100}$, &c. On the contrary, any decimal figure, by being removed one place toward the left, becomes ten times greater.

Equidistant

Equidistant places on the left and right of the place of units come under similar names, *viz.* tens and tenth parts, hundreds and hundredth parts, thousands and thousandth parts, &c.; as in the following table.

<i>&c.</i>
C Millions.
X Millions.
Millions.
C Thousands.
X Thousands.
Thousands.
Hundreds.
Tens.
Units. ———
Tenth parts.
Hundredth parts.
Thousandth parts.
X Thousandth parts.
C Thousandth parts.
Millionth parts.
X Millionth parts.
C Millionth parts.

&c.

An integer, by annexing ciphers, is raised to higher places on the left, and may by this means have its value increased to infinity. On the other hand, a decimal, by prefixing ciphers, is depressed to lower places on the right, and may by this means have its value diminished to infinity.

Ciphers annexed to decimals do not change the value of the decimals. Thus, $.50 = 5$, and $.500 = 5$, for $.50 = \frac{50}{100} = \frac{5}{10} = .5$; and $.500 = \frac{500}{1000} = \frac{5}{10} = .5$.

Decimals may be resolved into constituent parts, and the parts may be read, separately, thus. $.847 = 8 + .04 + .007 = \frac{8}{10} + \frac{4}{100} + \frac{7}{1000}$.

In decimals the figure next the point, being the first decimal place, is sometimes called *primes*, and the second figure from the point is called *seconds*, the next

A 2

thirds,

thirds, &c. Thus, in .875 the figure 8 is primes, 7 is seconds, and 5 is thirds.

From this brief account of the nature of decimals, it follows, that the manner of operation in decimals will be the same as in whole numbers; and also, that the same number may be differently expressed, according as the integer is chosen. Thus, the time since our Saviour's birth may be written thus, 1765; or thus, 176.5; or thus, 17.65; or thus, 1.765; or thus, .1765, according as one year, a decad, a century, a chiliad, or myriad, is used as the integer. Hence arises the superior excellency of decimal arithmetic, above every other sort of numerical computation; as will appear, with convincing evidence, in the sequel.

CH A P. II.

REDUCTION OF DECIMALS.

P R O B. I.

To reduce a vulgar fraction to a decimal.

R U L E.

To the numerator of the vulgar fraction affix a point or comma, then annex a competent number of ciphers, and divide by the denominator; the quot is the numerator of the decimal, and the ciphers annexed show the number of decimal places.

E X A M P L E I.

Reduce $\frac{1}{2}$ to a decimal?

Here to the numerator 1 I annex one cipher, and dividing by the denominator 2, the quot is 5, and 0 remains; and because a single cipher only was annexed to the numerator, the decimal numerator will consist but of one figure, namely 5; to which, therefore, I prefix the decimal point. So $\frac{1}{2} = .5$.

$$\begin{array}{r} 2 \overline{) 1.0(.5} \\ \underline{1 } \\ 0 \\ \underline{0} \end{array}$$

Hence

Hence appears the reason of the rule; namely $2:1::10:5$; that is, as the vulgar denominator to the vulgar numerator, so is the decimal denominator to the decimal numerator.

The truth of the rule will still further appear, if we consider, that the decimal fraction is produced by multiplying the numerator and denominator of the vulgar fraction by the same number, namely, the quot arising from the division of the decimal denominator by the vulgar denominator, or the quot of the decimal numerator divided by the vulgar numerator. Thus, $\frac{10}{2}$ or $\frac{5}{1} = 5$, and $\frac{1 \times 5}{2 \times 5} = \frac{5}{10} = .5$.

EXAMPLE II.

Reduce $\frac{3}{4}$ to a decimal.

To the numerator 3, I annex two ciphers; and, dividing by the denominator, the quot gives 75 for the numerator of the decimal, two ciphers having been annexed. So $\frac{3}{4} = .75$.

$$\begin{array}{r} 4 \overline{) 3.00} \quad (.75) \\ \underline{28} \\ 20 \\ \underline{20} \\ (0) \end{array}$$

EXAMPLE III.

Reduce $\frac{1}{16}$ to a decimal.

Here I annex four ciphers; and as the quot gives only three decimal places I supply the defect by prefixing a cipher; so $\frac{1}{16} = .0625$.

$$\begin{array}{r} 16 \overline{) 1.0000} \quad (.0625) \\ \dots \\ 96 \\ \underline{ 96} \\ 40 \\ 32 \\ \underline{ 32} \\ 80 \\ 80 \\ \underline{ 80} \\ (0) \end{array}$$

The reason of prefixing the cipher is obvious; for $\frac{625}{10000} = .0625$, by the manner of notation.

E X

EXAMPLE IV.

Reduce $\frac{7}{3125} =$ to a decimal.

Here I annex five ciphers; $3125)7.00000(.00224$
and as the quot gives only three
decimal places, I prefix two ci-
phers. So $\frac{7}{3125} = .00224$.

$$\begin{array}{r}
 6250 \\
 \hline
 7500 \\
 6250 \\
 \hline
 12500 \\
 12500 \\
 \hline
 (0)
 \end{array}$$

Though ciphers may be annexed at pleasure, yet it is the ciphers used that determine the number of decimal places in the quot; and at first it is sufficient to annex so many as serve to complete the first dividual, leaving room to annex more as you proceed in the operation; or, rather, annex the other ciphers to the remainders, without giving them a place in the dividend.

The first dividual also shows whether ciphers ought to be prefixed to the quot, and how many. Thus, if the first dividual take in only one of the annexed ciphers, the figure put in the quot is primes, and no cipher to be prefixed. If the first dividual comprehend two of the annexed ciphers, the figure put in the quot is seconds, and one cipher must be prefixed. If the first dividual comprehend three of the annexed ciphers, the figure put in the quot is thirds, and two ciphers must be prefixed, &c. Hence, in reducing a vulgar fraction to a decimal, the natural and easy way is, to place first the decimal point in the quot, and after it a cipher or ciphers, or the quotient-figure, as the first dividual directs.

In reducing a vulgar fraction to a decimal, if 0 at last remains, as in all the above examples, the decimal is precisely equal to the vulgar fraction, and is called a *finite or terminate decimal*.

In finite decimals, the denominator is always some aliquot part of the numerator increased by annexed ciphers; and such decimals take their rise from vulgar fractions

fractions whose denominator is 2 or 5, or some power of 2 or 5, or the product of some of their powers. — See powers described in the chapter on Extraction of Roots, in Part III.

The powers of numbers are sometimes expressed by indices or exponents placed at the corners of the numbers. Thus, 2^2 signifies the second power of 2, and 5^3 signifies the third power of 5; and 10^4 signifies the fourth power of 10, &c. The index of the root or first power is seldom expressed.

Any power of 2 multiplied into the like power of 5 gives a product equal to the same power of 10; as appears from the following specimen of the powers of 2, 5, and 10.

$2^1 = 2$	$5^1 = 5$	$2 \times 5 = 10$	$10^1 = 10$
$2^2 = 4$	$5^2 = 25$	$4 \times 25 = 100$	$10^2 = 100$
$2^3 = 8$	$5^3 = 125$	$8 \times 125 = 1000$	$10^3 = 1000$
$2^4 = 16$	$5^4 = 625$	$16 \times 625 = 10000$	$10^4 = 10000$
$2^5 = 32$	$5^5 = 3125$	$32 \times 3125 = 100000$	$10^5 = 100000$
$2^6 = 64$	$5^6 = 15625$	$64 \times 15625 = 1000000$	$10^6 = 1000000$
$2^7 = 128$	$5^7 = 78125$	$128 \times 78125 = 10000000$	$10^7 = 10000000$
&c.	&c.	&c.	&c.

The product of two different powers of 2 and 5, is equal to the product that will arise by raising 10 to the power denoted by the lesser given index, and then multiplying this power of 10 into that power of the other number which is denoted by the difference of the two given exponents. Thus,

$$2^6 \times 5^2 = 64 \times 25 = 10^2 \times 2^4 = 100 \times 16 = 1600$$

$$2^2 \times 5^6 = 4 \times 15625 = 10^2 \times 5^4 = 100 \times 625 = 62500$$

From these remarks it is easy to perceive, that 2 or 5, or any of their powers, or product of their powers, will measure 10 or its powers, viz. 100, 1000, &c. or their multiples, such as, 20, 200, 2000, &c. 30, 300, 3000, &c.; and such every numerator becomes by having ciphers annexed; and therefore 2 or 5, or their powers, or product of their powers, used as a denominator, will divide any numerator with a competent number of ciphers annexed, and leave no remainder;

remainder; and consequently the decimal thence resulting will be finite.

If the numerator of the vulgar fraction be unity, and the denominator any single power of 2 or 5, there will be as many decimal places in the quot as there are units in the index of the given power. Thus, $16 = 2^4$ gives a decimal of four places, viz. $\frac{1}{16} = .0625$; and, $125 = 5^3$ gives a decimal of three places, viz. $\frac{1}{125} = .008$.

When the denominator is the product of like powers of 2 and 5; in this case, such a product being equal to the like power of 10, and any power of 10 being equal to 1, with as many ciphers annexed as there are units in the index, it follows, that there will still be as many decimal places in the quot as there are units in the index, either of 2, of 5, or 10. Thus, $8 \times 125 = 2^3 \times 5^3 = 10^3 = 1000$, gives a decimal of three places, viz. $\frac{1}{1000} = .001$.

When the denominator is the product of different powers of 2 and 5, find what power of 10, and what power of 2 or 5, upon being multiplied, will give the same product, as is taught above; and the sum of the indices shews the number of decimal places; Thus, $2^6 \times 5^2 = 10^2 \times 2^4$; and the sum of the indices, $2 + 4 = 6$, gives the number of decimal places, viz. $\frac{1}{1600} = .000625$.

And, in general, to find what number of decimal places any such vulgar fraction will give, divide the denominator by 2, 5, or 10, till the last quotient be 1, and the remainder 0; and the number of divisors shews the number of decimal places. Thus, $\frac{1}{16}$ gives a deci-

mal of four places; for $2)16(8(4(2(1$. And $\frac{1}{125}$ gives

a decimal of three places; for $5)125(25(5(1$. And $\frac{1}{1000}$ gives a decimal of three places; for $10)1000(10(10(10(1$.

And $\frac{1}{1600}$ gives a decimal of six places; for $10)1600(160(16(8(4(2(1$.

for 10)1600(160(16(8(4(2(1.

If

If the denominator of a vulgar fraction be neither 2 nor 5, nor any of their powers, nor product of their powers, such a denominator will not divide the numerator with annexed ciphers without a remainder; and the decimal thence resulting is called *infinite*, or *interminate*.

Of infinite or interminate decimals, there are two sorts. For some constantly repeat the same figure; and are called *repeating decimals*, *repeaters*, or *single repetends*. Others repeat a circle of figures; and on that account are called *circulating decimals*, *circulates*, or *compound repetends*.

EXAMPLE V.

Reduce $\frac{1}{3}$ to a decimal.

3)1.0(.3

Here the remainder being still the same, *viz.* 1, the same figure will constantly be repeated in the quot.

9
(1)

In like manner, $\frac{2}{3} = .6$, and $\frac{1}{5} = .2$, $\frac{2}{5} = .4$, $\frac{3}{5} = .6$, $\frac{4}{5} = .8$; and, in general, the denominator 5 gives a continued repetition of the numerator.

Repeating decimals are of two kinds: *viz.* some consist only of the repeating figures, such as the examples above; and these are called *pure repeaters*: others have one or more digits or ciphers betwixt the decimal point and the repeating figure; and these are called *mixt repeaters*; and the digits or ciphers on the left of the repeating figures are called the *finite part* of such decimals.

Pure repeaters take their rise from vulgar fractions whose denominator is 3, or its multiple 9; and are but few in number.

Mixt repeaters derive their origin from vulgar fractions whose denominator is the product of 3 into 2 or 5, or into some of their powers, or product of their powers; and such denominators may be considered as the product of two component parts, whereof one is 2 or 5, or some of their powers, or product of their powers; and hence the finite part. The other component part is 3; and hence the repeating figure.

EXAMPLE VI.

Reduce $\frac{4}{15}$ to a decimal.

Here the repeater is mixt, the finite part being 2, and the repeating figure 6.

$$\begin{array}{r} 15 \overline{) 4.0(.26} \\ \underline{30} \\ * 100 \\ \underline{90} \\ (10) \end{array}$$

We may resolve such denominators into their component parts, and divide the numerator by one of these parts, and then divide the quot by the other. Thus, $15 = 5 \times 3$.

$$\begin{array}{r} 5 \overline{) 4.0(.8} \\ \underline{40} \\ (0) \end{array}$$

$$\begin{array}{r} \text{and } 3 \overline{) .8(.26} \\ \underline{6} \\ * 20 \\ \underline{18} \\ (2) \end{array}$$

The number of places in the finite part of a mixt repeater may be ascertained from the number of units in the index of the powers of 2 or 5, as appears plain in the following specimen.

$$\frac{1}{6} = \frac{1}{2 \times 3} = .16$$

$$\frac{1}{12} = \frac{1}{2^2 \times 3} = .083$$

$$\frac{1}{24} = \frac{1}{2^3 \times 3} = .0416$$

$$\frac{1}{48} = \frac{1}{2^4 \times 3} = .02083$$

$$\frac{1}{15} = \frac{1}{5 \times 3} = .06$$

$$\frac{1}{75} = \frac{1}{5^2 \times 3} = .013$$

$$\frac{1}{375} = \frac{1}{5^3 \times 3} = .0026$$

$$\frac{1}{1875} = \frac{1}{5^4 \times 3} = .00053$$

And, universally, to find the number of places in the finite part of such fractions, divide the denominator first by 3, and then divide the quot by 2, 5, or 10, till the last

last quot be 1, and 0 remain; and the number of divisors, excluding 3, shows the number of places in the finite part. Thus, $\frac{1}{48}$ gives four finite places; for 3)48(16,

2)2)2) and 2)16(8(4(2(1. And $\frac{1}{1875}$ gives also four finite places; for 3)1875(625, and 5)625(125(25(5(1.

Repeating decimals are usually marked by a dash through the right hand figure, as in all the examples above: but some chuse to mark them by a point set over the repeating figure, thus, $\dot{.3}$, $\dot{.26}$. The remainder where the repetition begins is commonly marked with an asterisk.

Because any quotient multiplied by the divisor reproduces the dividend, it follows, that any decimal multiplied by the denominator of the vulgar fraction from which it resulted, will reproduce the numerator with the annexed ciphers. Thus, if $.75$, the decimal of $\frac{3}{4}$, be multiplied by 4, it will reproduce the numerator 3 and the two annexed ciphers.

Now, suppose the given decimal to be a repeater; such as $.3$, resulting from the vulgar fraction $\frac{1}{3}$, if the repeating decimal be multiplied by the denominator 3, it will, by carrying at 9 on the right hand, reproduce the numerator 1 with the annexed cipher. In like manner, if the repeater $.6 = \frac{2}{3}$, be multiplied by 3, it will, by carrying at 9 on the right hand, reproduce the numerator 2 with the annexed cipher. Again, if the repeater $.1 = \frac{1}{9}$, be multiplied by the denominator 9, it will, by carrying at 9 on the right hand, reproduce the numerator 1 with the annexed cipher. And, if the mixt repeater $.26 = \frac{4}{15}$, be multiplied by the denominator 15, it will, by carrying at 9 on the right hand, reproduce the numerator 4 with the two annexed ciphers.

From these remarks we may conclude, that the right-hand figure of every repeating decimal is ninth-parts: and the same truth may be evinced by resolving the decimal into its constituent parts, in the following manner.

The vulgar fraction $\frac{7}{9}$ reduced to a decimal gives $.77, \text{ \&c.}$; and this repeater resolved into decimal consti-

tuent parts, becomes $\frac{7}{10} + \frac{7}{100} + \frac{7}{1000}$, &c. to infinity. But if we esteem the right-hand figure to be ninth-parts, we have $\frac{7}{10} + \frac{7}{9}$ of $\frac{1}{10} = \frac{7}{10} + \frac{7}{90} = \frac{630}{900} + \frac{70}{900} = \frac{700}{900} = \frac{7}{9}$, the given vulgar fraction. And as the vulgar fraction $\frac{7}{9}$ gives .77, so $\frac{8}{9}$ gives .89; that is, $\frac{8}{9} = .89 = 1$. And, universally, a series of nines infinitely continued is equal to unity in the place on the left hand; Thus, .999 = 1; and .0999 = .1; and .0099 = .01. and 44.99 = 45.

Hence may be ascertained the value of an infinite series decreasing in a decuple proportion. Thus, $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$, &c. = $\frac{1}{9}$. And $\frac{2}{10} + \frac{2}{100} + \frac{2}{1000}$, &c. = $\frac{2}{9}$.

If the denominator of a vulgar fraction be neither 2 nor 5, nor any power of 2 or 5, nor any product of their powers; nor 3, nor 9, nor any product of 3 into 2 or 5, or into some of their powers, or product of their powers, the decimal resulting from such a vulgar fraction will circulate.

Circulates, like repeaters, are of two sorts, viz. pure and mixt. A pure circulate consists of the figures of the circle only; as, .09, 09, &c. or 18, 18, &c. A mixt circulate has a finite part betwixt the decimal point and the figure that begins the circle; as, .0, 45, 45, &c.; or .32, 142857, 142857, &c. Some chuse to distinguish the finite part from the circle, and one circle from another, by a comma, as above. Others dash the first and last figure of the circle. It is likewise usual to mark the remainder where the new circle begins, by affixing an asterisk.

EXAMPLE VII.

Reduce $\frac{1}{11}$ to a decimal.

The denominator 11 gives a pure 11)1.00(.09,09, circle of two figures.

$$\begin{array}{r} 99 \\ \hline * 100 \\ 99 \\ \hline * 1 \end{array}$$

$$\frac{2}{11} =$$

$\frac{2}{11} = .18, 18,$	$\frac{7}{11} = .63, 63,$
$\frac{3}{11} = .27, 27,$	$\frac{8}{11} = .72, 72,$
$\frac{4}{11} = .36, 36,$	$\frac{9}{11} = .81, 81,$
$\frac{5}{11} = .45, 45,$	$\frac{10}{11} = .90, 90,$
$\frac{6}{11} = .54, 54,$	

It is easy to perceive, that if any of the vulgar fractions in the above specimen have both its numerator and denominator multiplied by 9, there will arise a new vulgar fraction of the same value, whose numerator will be the figures of the circle, and its denominator the like number of 9's. Thus,

$$\frac{3 \times 9}{11 \times 9} = \frac{27}{99}, \text{ and } \frac{7 \times 9}{11 \times 9} = \frac{63}{99}.$$

As the denominator 11, whereof 99 is a multiple, gives a pure circulate of two places, so any denominator, whereof 999, or 9999, or 99999, &c. are multiples, will give a pure circulate of three, four, five, &c. places; that is, of as many places as there are 9's in the multiple. And such denominators are all the prime numbers, except 2, 3, and 5, viz. 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, &c.; also their products into 3, viz. 21, 33, 39, 51, 57, 69, &c. Such too are all the powers of 3, except 3 and 9, viz. 27, 81, 243, 729, 2187, &c.

The reason is plain: for if any divisor, as 37, divide 999 without a remainder, it will also divide 1000, and leave a remainder of 1, to begin a new circle.

To find how many places the circle will consist of, divide a competent number of 9's by any of the above denominators, continuing the operation till 0 remain; and the number of 9's used will show the number of places.

Thus, $7 \overline{)999999}$ six places. Thus, $27 \overline{)999}$ three places.

142857

37

The

tuent parts, becomes $\frac{7}{10} + \frac{7}{100} + \frac{7}{1000}$, &c. to infinity. But if we esteem the right-hand figure to be ninth-parts, we have $\frac{7}{10} + \frac{7}{9}$ of $\frac{1}{10} = \frac{7}{10} + \frac{7}{9} = \frac{630}{900} + \frac{700}{900} = \frac{1330}{900} = \frac{133}{90}$, the given vulgar fraction. And as the vulgar fraction $\frac{7}{9}$ gives .77, so $\frac{2}{9}$ gives .22; that is, $\frac{2}{9} = 1$. And, universally, a series of nines infinitely continued is equal to unity in the place on the left hand; Thus, .999 = 1; and .0999 = .1; and .0099 = .01. and 44.99 = 45.

Hence may be ascertained the value of an infinite series decreasing in a decuple proportion. Thus, $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$, &c. = $\frac{1}{9}$. And $\frac{2}{10} + \frac{2}{100} + \frac{2}{1000}$, &c. = $\frac{2}{9}$.

If the denominator of a vulgar fraction be neither 2 nor 5, nor any power of 2 or 5, nor any product of their powers; nor 3, nor 9, nor any product of 3 into 2 or 5, or into some of their powers, or product of their powers, the decimal resulting from such a vulgar fraction will circulate.

Circulates, like repeaters, are of two sorts, viz. pure and mixt. A pure circulate consists of the figures of the circle only; as, .09, 09, &c. or 18, 18, &c. A mixt circulate has a finite part betwixt the decimal point and the figure that begins the circle; as, .0, 45, 45, &c.; or .32, 142857, 142857, &c. Some chuse to distinguish the finite part from the circle, and one circle from another, by a comma, as above. Others dash the first and last figure of the circle. It is likewise usual to mark the remainder where the new circle begins, by affixing an asterisk.

EXAMPLE VII.

Reduce $\frac{1}{11}$ to a decimal.

The denominator 11 gives a pure 11)1.00(.09,09, circle of two figures.

$$\begin{array}{r} 99 \\ \hline * 100 \\ 99 \\ \hline * 1 \end{array}$$

$$\frac{2}{11} =$$

$$\begin{array}{l|l}
 \frac{2}{11} = .18, 18, & \frac{7}{11} = .63, 63, \\
 \frac{3}{11} = .27, 27, & \frac{8}{11} = .72, 72, \\
 \frac{4}{11} = .36, 36, & \frac{9}{11} = .81, 81, \\
 \frac{5}{11} = .45, 45, & \frac{10}{11} = .90, 90, \\
 \frac{6}{11} = .54, 54, &
 \end{array}$$

It is easy to perceive, that if any of the vulgar fractions in the above specimen have both its numerator and denominator multiplied by 9, there will arise a new vulgar fraction of the same value, whose numerator will be the figures of the circle, and its denominator the like number of 9's. Thus,

$$\frac{3 \times 9}{11 \times 9} = \frac{27}{99}, \text{ and } \frac{7 \times 9}{11 \times 9} = \frac{63}{99}.$$

As the denominator 11, whereof 99 is a multiple, gives a pure circulate of two places, so any denominator, whereof 999, or 9999, or 99999, &c. are multiples, will give a pure circulate of three, four, five, &c. places; that is, of as many places as there are 9's in the multiple. And such denominators are all the prime numbers, except 2, 3, and 5, viz. 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, &c.; also their products into 3, viz. 21, 33, 39, 51, 57, 69, &c. Such too are all the powers of 3, except 3 and 9, viz. 27, 81, 243, 729, 2187, &c.

The reason is plain: for if any divisor, as 37, divide 999 without a remainder, it will also divide 1000, and leave a remainder of 1, to begin a new circle.

To find how many places the circle will consist of, divide a competent number of 9's by any of the above denominators, continuing the operation till 0 remain; and the number of 9's used will show the number of places.

Thus, $7 \overline{)999999}$ six places. Thus, $27 \overline{)999}$ three places.

142857

37

The

The number of figures in a circle, when some power of 3 is the denominator, may also be found thus: Divide the given denominator by 9, and the number of units in the quot will be equal to the number of figures in the circle. Thus, $9)27$ (3 places. Thus, $9)81$ (9 places. Thus, $9)729$ (81 places. Thus, $9)2187$ (243 places, &c.

I shall now subjoin a further specimen of the pure circulates that arise from the above primes, their products into 3, and powers of 3, and make a few remarks on them.

	Places.
$\frac{1}{7} = .142857, 142857, 142857, 142857, 14$	six
$\frac{1}{11} = .09, 09, 09, 09, 09, 09, 09, 09, 09, 09, 09,$	two
$\frac{1}{13} = .076923, 076923, 076923, 076923, 07$	six
$\frac{1}{17} = .0588235294117647, 0588235294$	sixteen
$\frac{1}{19} = .052631578947368421, 05263157$	eighteen
$\frac{1}{21} = .047619, 047619, 047619, 047619, 04$	six
$\frac{1}{23} = .0434782608695652173913, 0434$	twenty-two
$\frac{1}{27} = .037, 037, 037, 037, 037, 037, 037,$	three
$\frac{1}{29} = .0344827586206896551724137931, 03$	twenty-eight
$\frac{1}{31} = .032258064516129, 032258064$	fifteen
$\frac{1}{33} = .03, 03, 03, 03, 03, 03, 03, 03, 03, 03, 03,$	two
$\frac{1}{37} = .027, 027, 027, 027, 027, 027, 027, 027,$	three
$\frac{1}{39} = .025641, 025641, 025641, 025641, 02$	six
$\frac{1}{41} = .02439, 02439, 02439, 02439, 02439,$	five
$\frac{1}{43} = .023255813953488372093, 023$	twenty-one

The greatest possible number of places in a circle will always be one less than the number of units in the prime denominator. Thus, 7 dividing any numerator may leave a remainder of 1, 2, 3, 4, 5, or 6, but cannot leave 7; and therefore the number of places in the resulting circle cannot exceed six: for where any former remainder recurs, the circle begins anew.

Of the above circulates, those resulting from the prime numbers,

numbers, and consisting of an even number of places, have the sum of their figures equal to the product of 9 into half the number of figures in the circle. Thus, $.142857, = 27 = 9 \times 3$. And, $.09 = 9 = 9 \times 1$. And, $.0588235294117647, = 72 = 9 \times 8$. For one half of the circle, added to the other half, gives a series of 9's, equal in number to half the figures of the circle.

Thus, 142

857

999

and thus, 05882352

94117647

99999999

If 3 divide a repeater whose repeating figure is not a multiple of 3, the quot will be a pure circulate of three places. Thus, $3) .\dot{1}11(.037$, and $3) .\dot{5}55(.185$, and $3) .\dot{7}77(.259$,

If 3 divide a pure circulate, the circle not being a multiple of 3, the quot will be a pure circulate of thrice as many places as the circle of the dividend. Thus, $3) .037, 037, 037, (.012345679$,

Mixt circulates take their rise from fractions whose denominators are the prime numbers 7, 11, 13, 17, 19, 23, 29, &c. multiplied into 2, 5, or 10, or into some of their powers, or product of their powers.

EX.

EXAMPLE VIII.

Reduce $\frac{2}{28}$ to a decimal.

The denominator $28 = 7 \times 2 \times 2$, gives a mixt circulate, consisting of the finite part 32, and a circle of six figures or places, whose sum is equal to the product of 9 into half the number of figures; that is, $9 \times 3 = 27$.

$$\begin{array}{r}
 28 \overline{) 9.0(32,142857,14} \\
 \underline{84} \\
 60 \\
 \underline{56} \\
 *40 \\
 \underline{28} \\
 120 \\
 \underline{112} \\
 80 \\
 \underline{56} \\
 240 \\
 \underline{224} \\
 160 \\
 \underline{140} \\
 200 \\
 \underline{196} \\
 *40 \\
 \underline{28} \\
 120 \\
 \underline{112} \\
 8
 \end{array}$$

A further specimen of mixt circulates, for the learner's instruction, is here subjoined.

$$\frac{1}{14} =$$

$$\frac{1}{7} = \frac{1}{7 \times 2} = .0,714285,714285,714285,$$

$$\frac{3}{22} = \frac{3}{11 \times 2} = .1,36,36,36,36,36,36,36,$$

$$\frac{49}{82} = \frac{49}{13 \times 2 \times 2} = .94,230769,230769,230769,$$

$$\frac{67}{85} = \frac{67}{17 \times 5} = .7,8823529411764705,882$$

$$\frac{463}{478} = \frac{463}{19 \times 5 \times 5} = .97,473684210526315789,47$$

$$\frac{278}{390} = \frac{278}{39 \times 10} = .7,128205,128205,128205,$$

$$\frac{18254}{20500} = \frac{18254}{41 \times 10 \times 10 \times 5} = .890,43902,43902,$$

The number of places, both in the finite part and in the circle, may be ascertained thus: Divide the denominator of the vulgar fraction by 10, 5, or 2, as often as possible, and the number of divisors will show the number of places in the finite part; make the last quot a divisor, and the dividend any competent number of 9's; continue the operation till 0 remain, and the number of 9's used will be equal to the number of places in the

circle. Thus, 10)20500(2050(205(41; and 41)99999(2439, and 0 remains. So I conclude, that the finite part will consist of three places, and the circle of five.

Universally, any vulgar fraction being given, we may determine whether the decimal thence resulting will be finite or infinite; and if infinite, whether pure or mixt; with the number of places, &c. in the following manner.

Reduce the given vulgar fraction to its lowest terms, then divide the denominator by 10, 5, or 2, as often as possible; and if the last quot be unity, without any re-

mainder, the decimal is finite, and the number of divisors shews the number of decimal places. Thus, $\frac{18}{980}$

reduced to its lowest terms is $\frac{3}{160}$; and $10)160(16(8$

places, viz. .01875.

If the last quot be 3, or any power of 3, the resulting decimal will be a mixt repeater, the number of whose finite places will be equal to the number of divisors. Thus, suppose the vulgar fraction in its lowest terms to

be $\frac{7}{24}$; now, $2)24(12(6(3$; and accordingly the resulting decimal is a mixt repeater, whose finite part consists of three places, namely .2916.

If the last quot cannot be divided by 2, 5, 10, or 3, the resulting decimal will be a mixt circulate; and the way of finding the number of places, both in the finite part and circle, is taught above.

If the denominator of the given vulgar fraction can be divided, neither by 2, 5, nor 10, the resulting decimal will be a pure repeater, or a pure circulate, according as the denominator is 3 or 9, or some of the prime numbers 7, 11, 13, &c.; as has been already explained.

Every vulgar fraction may be reduced to a decimal, finite or infinite; that is, to a finite decimal, to a repeater, or a circulate. For if the denominator divide the numerator with ciphers annexed, so as to leave no remainder, the resulting decimal is finite. If the remaining figure be always the same, the resulting decimal will be a repeater. If neither of these be the case, yet, because the divisor is a finite number, the remainder at last must either be the same with the numerator of the vulgar fraction, or the same with some preceding remainder, and then a new circle begins; and consequently the resulting decimal will be a circulate.

Because in circulates the circle runs on sometimes to 16, 18, 22, 28, 81, 243, &c. places, and because, in decimals of every sort, the finite part runs sometimes on to many places, such circulates, or finite parts, may, with-

out

out any sensible error, be limited at five or six places, and used as finites : for five decimal places, divide the integer into 100,000 equal parts, and all the loss that can be occasioned by such limitation is less than one hundred thousandth part of the integer. And in most cases, the decimal may be limited at three places, which divide the integer into 1000 equal parts.

Circulates, or finite parts, thus limited, are called *approximate decimals*; and are sometimes marked with + or — annexed, according as the right-hand figure is taken less or greater than just : for in limiting the decimal, if you foresee that the succeeding figure of the quot would be 6 or 7, or any figure above 5, you lessen the error by increasing the right-hand figure of the approximate by unity.

P R O B. II.

To reduce the parts of Coin, Weight, Measure, Time, &c. to decimals.

This may be done several ways, as follows.

M E T H O D I.

Convert the given part or parts to a vulgar fraction of the integer, and then reduce the vulgar fraction to a decimal.

I. M O N E Y.

Ex. i. Reduce 9 pence to the decimal of a shilling.

d. s.

$$9 = \frac{9}{12}$$

and 12)9.0(.75 of a shilling.

84

60

60

(0)

Here the fraction $\frac{9}{12} = \frac{3}{4}$; and the denominator $4 = 2 \times 2$ gives a finite decimal of two places.

C-2

Ex.

Ex. 2. Reduce 9 pence to the decimal of a pound.

d. L.

$$9 = \frac{9}{240}$$

$$240) 9.00(.0375 \text{ of a pound.}$$

7 20

1 800

1 680

1 200

1 200

(0)

The fraction $\frac{9}{240} = \frac{3}{80}$;
and the denominator $80 =$
 $10 \times 2 \times 2 \times 2$ gives a finite
decimal of four places.

Ex. 3. Reduce 16 s. 6 d. to the decimal of a pound.

s. d. L.

$$16 \quad 6 = \frac{198}{240}$$

$$240) 198.0(.825 \text{ L.}$$

12

198

1920

600

480

1200

1200

(0)

The fraction $\frac{198}{240} = \frac{33}{40} = \frac{33}{40}$;
and the denominator $40 = 10 \times 2$
 $\times 2$ gives a finite decimal of three
places.

Ex. 4. Reduce 18 s. 4 d. to the decimal of a pound.

s. d. L.

$$18 \quad 4 = \frac{220}{240} = \frac{11}{12}$$

$$12) 11.0(.91\bar{6} \text{ L.}$$

12

220

108

20

12

80

72

(8)

The denominator $12 = 2 \times 2 \times 3$
gives a mixt repeater, the finite part
consisting of two places.

More examples, one pound being the integer.

Ex.	s.	d.	f.	L.	L.
5.	19	11	3	$= \frac{252}{256}$	$= .9989583$
6.	15	6	1	$= \frac{745}{960}$	$= .776041\bar{6}$

Ex.

Ex.	s.	d.	f.	L.	L.
7.	14	3	2	$= \frac{686}{960} =$	$.71458\bar{3}$
8.	10	6	0	$= \frac{21}{40} =$	$.525$
9.	12	8	1	$= \frac{609}{960} =$	$.634375$
10.	13	4	0	$= \frac{2}{3} =$	$.6$
11.	6	8	0	$= \frac{1}{3} =$	$.3$
12.	0	7	0	$= \frac{7}{240} =$	$.0291\bar{6}$
13.	0	0	1	$= \frac{1}{960} =$	$.001041\bar{6}$

2. AVOIRDUPOIS WEIGHT.

Ex. 1. Reduce 17 lb. to the decimal of a quarter.

$$\text{lb. } 17 = \frac{17}{28}$$

$$28 \overline{) 17.0} \begin{matrix} .60,714285,71 \end{matrix}$$

168

* 200

196

40

28

120

112

80

56

240

224

160

140

* 200

196

40

28

(12)

Here the denominator 28, being the product of the prime number $7 \times 2 \times 2$, gives a mixt circulate, the finite part consisting of two places, and the circle of six figures; for $7)999999(142857$, and leaves no remainder.

Ex.

Ex. 2. Reduce 3 Q, 15 lb. to the decimal of a C.

$$\begin{array}{r}
 \text{Q. lb. C.} \\
 3 \text{ } 15 = \frac{29}{112} \\
 28 \\
 \hline
 99
 \end{array}
 \quad
 \begin{array}{r}
 112 \overline{) 99.0} \begin{array}{l} .8839,285714,28 \text{ C.} \\ 896 \\ \hline 940 \\ 896 \\ \hline 440 \\ 336 \\ \hline 1040 \\ 1008 \\ \hline 320 \\ 224 \\ \hline 960 \\ 896 \\ \hline 640 \\ 560 \\ \hline 800 \\ 784 \\ \hline 160 \\ 112 \\ \hline 480 \\ 448 \\ \hline 320 \\ 224 \\ \hline 960 \\ 896 \\ \hline (64) \end{array}
 \end{array}$$

The denominator 112, being the product of the prime number $7 \times 2 \times 2 \times 2 \times 2$, gives a mixt circulate; the finite part consisting of four places, and the circle of six figures.

More examples, one C. or 112 lb. being the integer.

$$\begin{array}{l}
 \text{Ex. Q. lb. oz. C. C.} \\
 3 \text{ } 1 \text{ } 27 \text{ } 15 = \frac{895}{1792} = .49944196,428571,42 \\
 4 \text{ } 3 \text{ } 14 \text{ } 0 = \frac{7}{8} = .875
 \end{array}$$

4 WINE-MEASURE.

Ex. 1. Reduce 3 pints to the decimal of a gallon.

Pts. gall.

$$3 = \frac{3}{8}$$

$$8) 3.0(.375 \text{ gall.}$$

$$\underline{24}$$

The denominator $8 = 2 \times 2 \times 2$ gives
a finite decimal of three places.

$$\underline{60}$$

$$\underline{56}$$

$$\underline{40}$$

$$\underline{40}$$

$$(0)$$

Ex. 2. Reduce 25 gallons 7 pints to the decimal of a hoghead.

Gall. pts. hhd.

$$25 \text{ } 7 = \frac{257}{304} = \frac{69}{168} = \frac{23}{56}$$

$$56) 23.0(.410,714285,7 \text{ hhd.}$$

$$\underline{207}$$

$$\underline{224}$$

$$\underline{60}$$

$$\underline{56}$$

$$* \underline{400}$$

$$\underline{392}$$

$$\underline{80}$$

$$\underline{56}$$

$$\underline{240}$$

$$\underline{224}$$

$$\underline{160}$$

$$\underline{112}$$

$$\underline{480}$$

$$\underline{448}$$

$$\underline{320}$$

$$\underline{280}$$

$$* \underline{400}$$

$$\underline{392}$$

$$(8)$$

The denominator $56 = 7 \times 2 \times 2 \times 2$ gives a mixt circulate ;
the finite part consisting of three
places, and the circle of six fi-
gures.

More

More examples, one hog shead being the integer,

Ex. Gall. pts. Hhd.
 3. 62 7 = .998,015873,0:
 4. 33 5 = .53

5. LONG MEASURE.

Ex. 1. Reduce 5 inches to the decimal of a foot.

Inch. f.
 $5 = \frac{5}{12}$

$$\begin{array}{r} 12 \overline{) 5.0} \cdot 41\bar{6} \text{ foot.} \\ \underline{48} \\ 20 \\ \underline{12} \\ *80 \\ \underline{72} \\ (8) \end{array}$$

The denominator 12 = 2 × 2 × 3 gives a mixt repeater, as was already observed.

Ex. 2. Reduce 1 foot 8 inches to the decimal of a yard.

F. in.
 $1 \text{ } 8 = \frac{20}{36} = \frac{10}{18} = \frac{5}{9}$
 $\frac{12}{20}$

$$\begin{array}{r} 9 \overline{) 5.0} \cdot 5 \text{ yard.} \\ \underline{45} \\ (5) \end{array}$$

The denominator 9 gives a pure repeater, the quot being a continued repetition of the numerator.

More examples, one yard being the integer.

Ex. F. in. Yd.
 3. 2 11 = .977
 4. 1 9 = .583

6. TIME

Ex. 1. Reduce 17 hours to the decimal of a day.

H. Day.

$$17 = \frac{17}{24}$$

$$24)17.0(.708\bar{3} \text{ day.}$$

168

The denominator $24 = 2 \times 2 \times 2$
 $\times 3$ gives a mixt repeater, the finite
 part consisting of three places.

200

192

*80

72

(8)

Ex 2. Reduce 12 hours 36 minutes to the decimal of a day.

H. min. Day.

$$12 \ 36 = \frac{756}{1440} = \frac{378}{720} = \frac{63}{120} = \frac{21}{40}$$

60

$$40)21.0(.525 \text{ day.}$$

200

756

The denominator $40 = 10 \times 2 \times 2$
 gives a finite decimal of three places.

100

80

200

200

(0)

More examples, one day being the integer.

Ex. H. min. Day.

$$3 \ 23 \ 59 = .99930\bar{8}$$

$$4 \ 5 \ 45 = .23958\bar{3}$$

METHOD II.

Here there are two cases.

i. If the given parts consist of one denomination, in this case reduce the given parts to a decimal of the next superior denomination; again, reduce this decimal to another decimal of the next superior denomination; and

and proceed in this manner till you bring it to a decimal of the integer required.

A decimal of a lower denomination is reduced to a decimal of a higher, if you divide it by the number of units in the lower which makes an unit of the higher. Thus, the decimal of a farthing is reduced to the decimal of a penny, by dividing it by 4; and the decimal of a penny is reduced to the decimal of a shilling, by dividing it by 12; and the decimal of a shilling is reduced to the decimal of a pound, by dividing it by 20.

Thus, let it be required to reduce 9 d. to the decimal of a pound Sterling. To perform this, I first reduce 9 d. to the decimal of a shilling, by dividing by 12; and then divide the decimal of a shilling by 20, as follows.

$$\begin{array}{r}
 \begin{array}{cc} d. & s. \end{array} & & 20) L. & d. \\
 9 = \frac{9}{12}, \text{ and } 12) 9.0 & & (.75(.0375 = 9 & \\
 & & \begin{array}{r} 84 \quad 60 \\ \hline 60 \quad 150 \\ 60 \quad 140 \\ \hline (0) \quad 100 \\ 100 \\ \hline (0) \end{array}
 \end{array}$$

In dividing by 20, because the first dividial takes in two decimal places, I set 0 after the decimal point, the quotient-figure 3 being decimal seconds.

2. If the given parts be of different denominations, work by the following

R U L E S.

1. Place the numbers that make an unit of their respective superior denominations, under one another, as so many divisors; setting those of the lowest denomination uppermost.

2. On the right of the divisors place the several given parts, as so many dividends, and the lowest of the parts uppermost.

3. Annex ciphers to the uppermost dividend; or, rather, suppose ciphers annexed; then divide, placing the quot on the right of the following dividend. Again, divide this mixt number by the following divisor, plac-

D 2

cing

cing the quot on the right of the subsequent dividend. Proceed in the same manner, till you have divided by every divisor, and the last quot is the decimal required.

EXAMPLE I.

Reduce 14 s. 9 d. 3 f. to the decimal of a pound.

Here the numbers that make an unit of their respective superior denominations, viz. 4, 12, 20, are placed under one another, as so many divisors; and opposite to them the given parts 3 f. 9 d. 14 s. are placed under one another, as dividends; and the several divisions being finished, the last quot .740625 is a decimal of a pound equal to 14 s. 9 d. 3 f.

$$\begin{array}{r|l}
 4 & 3 \\
 \hline
 12 & 9.75 \\
 \hline
 20 & 14.8125 \\
 \hline
 & .740625 \text{ L.}
 \end{array}$$

The reason of the rules is obvious: for the divisors are the component parts of 960, the number of farthings in a pound Sterling; and therefore, if we divide by these component parts, as in the above example, we shall have the same decimal as would be obtained by reducing the given parts to farthings, and then dividing by 960.

Ciphers on the right of any divisor may be cut off, as in the above example; or quite omitted, as in the following examples: such omission being compensated by the ciphers annexed to the dividend.

When the quot of any division is a repeater, or circulate, the next division must be carried on, not by annexing ciphers, but by annexing the repeated figure or figures of the circle; as in the examples following.

EXAMPLE II.

Reduce 18 s. 4 d. 1 f. to the decimal of a pound.

Here, in dividing by 12, the quot is a mixt repeater; and therefore, in dividing by 20, that is, by 2, after the dividend is exhausted, I annex the repeating figure 6, and the quot, or decimal sought is a mixt repeater.

$$\begin{array}{r|l}
 4 & 1 \\
 \hline
 12 & 4.25 \\
 \hline
 2 & 18.3541\overline{6} \\
 \hline
 & .917708\overline{3} \text{ L.}
 \end{array}$$

E X-

EXAMPLE III.

Reduce 3 Q. 18 lb. to the decimal of a C.

Here, because 28 is a troublesome divisor, I divide by the component parts 4 and 7, and the quot is a mixt circulate.

$$\begin{array}{r} 28 \left\{ \begin{array}{l} 4 \overline{) 18} \\ 7 \overline{) 4.5} \\ 4 \overline{) 3.6, 428571, 42} \\ \hline .910, 714285, 71 \end{array} \right. \end{array}$$

In dividing the circulate by 4, after the dividend is exhausted, instead of annexing ciphers, I annex the figures of the circle, and the quot, or decimal sought, comes out another mixt circulate.

EXAMPLE IV.

Reduce 10 oz. 18 dw. 16 gr. to the decimal of a lb. Troy.

Here, again, instead of dividing by 24, I divide by the component parts 4 and 6.

$$\begin{array}{r} 24 \left\{ \begin{array}{l} 4 \overline{) 16} \\ 6 \overline{) 4} \\ 2 \overline{) 18.6} \\ 12 \overline{) 10.93} \\ \hline 10.93 \end{array} \right. \end{array}$$

The examples left, in Method 1. for the learner's exercise, may be here resumed, to be reduced by this method.

METHOD II.

When the given parts consist of several denominations, we may find the decimal to each of these denominations separately; and the sum of the decimals thus found is the decimal required.

EXAMPLE I.

Reduce 16 s. 6 d. to the decimal of a pound.

$$\begin{array}{rcl} & L. & L. \\ 16 \text{ s.} & = \frac{16}{20} & = .8 \\ 6 \text{ d.} & = \frac{6}{240} & = .025 \quad s \quad d. \\ & & \hline & .825 & = 16 \quad 6 \end{array}$$

EX.

EXAMPLE II.

Reduce 13 s. 4 d. 2 f. to the decimal of a pound.

$$\begin{array}{rcl}
 & L. & L. \\
 13 \text{ s.} & = \frac{13}{20} & = .65 \\
 4 \text{ d.} & = \frac{4}{240} & = .016666 \\
 2 \text{ f.} & = \frac{2}{960} & = .002083 \\
 \hline
 & & .66875 = 13 \quad 4 \quad 2
 \end{array}
 \quad \begin{array}{l} s. \quad d. \quad f. \end{array}$$

EXAMPLE III.

Reduce 8 oz. 19 dw. 8 gr. to the decimal of a lb. Troy.

$$\begin{array}{rcl}
 & lb. & lb. \text{ Troy.} \\
 8 \text{ oz.} & = \frac{8}{12} & = .6666 \\
 19 \text{ dw.} & = \frac{19}{240} & = .0791\bar{6} \\
 8 \text{ gr.} & = \frac{8}{5760} & = .0013\bar{8} \\
 \hline
 & & .7472\bar{7} = 8 \quad 19 \quad 8
 \end{array}
 \quad \begin{array}{l} oz. \quad dw. \quad gr. \end{array}$$

The same answer may also be found by adding the vulgar fractions, and then reducing the vulgar fraction of their sum to a decimal.

$$\begin{aligned}
 \text{Thus, in Ex. 1. we may say, } \frac{16}{20} + \frac{6}{240} &= \frac{3840}{4800} + \frac{120}{4800} = \frac{3960}{4800} = \frac{396}{480} = \frac{132}{160} = \frac{66}{80} = \frac{33}{40} = .825.
 \end{aligned}$$

METHOD IV.

The decimal of given parts may frequently be known by inspection, without any operation: for because $\frac{1}{4} = .25$, $\frac{1}{2} = .5$, $\frac{3}{4} = .75$, $\frac{1}{3} = .3$, $\frac{2}{3} = .6$, $\frac{1}{5} = .2$, $\frac{1}{6} = .1\bar{6}$, $\frac{2}{6} = .3\bar{3}$, &c. it follows, that if the given parts be equal to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, &c. of the integer, the correspondent decimal will be .25, .5, .75, &c.

Thus, 3 inches $= \frac{1}{4}$ foot, $= .25$; and 6 inches $= \frac{1}{2}$ foot, $= .5$; and 9 inches $= \frac{3}{4}$ foot, $= .75$; and 4 inches $= \frac{1}{3}$ foot, $= .3$; and 8 inches $= \frac{2}{3}$ foot, $= .6$.

In like manner, 4 s. $= \frac{1}{5}$ L. $= .2$; and 6 s. 8 d. $= \frac{1}{3}$ L. $= .3$; and 13 s. 4 d. $= \frac{2}{3}$ L. $= .6$.

But the decimal answering to any number of shillings, pence, and farthings, may be readily found by inspection, to three places, in the following manner, viz.

Half

Half the greatest even number of shillings gives the figure for the place of primes; the number of farthings in the given pence and farthings, increased by unity, when their sum amounts to 24, or upwards, generally affords two figures more for seconds and thirds; but if the number of farthings in the given pence and farthings amount only to a single digit, that digit goes to the place of thirds, and 0 fills the place of seconds. Here observe, if the given number of shillings be odd, the place of seconds must be increased by 5.

EXAMPLES.

Ex. s. d. f. L.

1. 10 6 3 = .528

2. 14 1 1 = .705

3. 19 7 3 = .982

4. 17 6 0 = .875

5. 1 0 0 = .05

6. 0 2 1 = .009

In Example 1. the half of 10 s. gives 5 for primes; the 6 d. 3 f. make 27 farthings; which amounting to 24, I increase by unity, and have 28 for seconds and thirds.

In Example 2. the half of 14 s. gives 7 for primes, and 1 d. 1 f. make 5 farthings; which being a single digit, is so many thirds, and I put 0 in the place of seconds.

In Example 3. the half of 19 s. gives 9 for primes, and 7 d. 3 f. make 31 farthings; which amounting to 24, I increase by unity, and have 32; but 19 s. being an odd number, I increase the place of seconds by 5, and so have 82 for seconds and thirds.

In Example 4. the half of 17 s. gives 8 for primes, and 6 d. make 24 farthings; which, increased by unity, is 25; and because 17 s. is an odd number, I increase the place of seconds by 5, and so have 75 for seconds and thirds.

In

In Example 5. the half of 1 s. gives 0 for primes; and 1 s. being an odd number, gives 5 for seconds; and as there are no pence or farthings, there can be no thirds.

In Example 6. no shillings gives 0 for primes, and 2 d. 1 f. make 9 farthings; which, being a single digit, goes to the place of thirds, and 0 fills the place of seconds.

The reason of increasing the number of farthings by unity, when their sum amounts to 24, or upwards, is obvious: for a pound in farthings is 960, and in thousandth parts is 1000; consequently, $500 = 480$, and $50 = 48$, and $25 = 24$. If therefore to every 24 farthings unity be added, we change the farthings, or 960 parts, into 1000th parts.

The reason of adding 5 to the place of seconds for every odd shilling is also evident: for $1 \text{ s.} = \frac{1}{20} \text{ L.} = \frac{5}{100} = .05$

A decimal of three places is exact enough in most computations; but if the complete decimal be required, it may be found as follows.

Take half the number of farthings in the pence and farthings, rejecting sixpence if there be one; to the half number of farthings annex ciphers, or rather suppose ciphers annexed; divide by 12 till 0 remain, or till the figure in the quot repeat; and the quot annexed to the three figures formerly found is the decimal complete. The former six examples, completed in this manner, follow.

Ex.	s.	d.	f.	
1.	10	6	3	$= .528125$
2.	14	1	1	$= .705208\bar{3}$
3.	19	7	3	$= .982291\bar{6}$
4.	17	6	0	$= .875$
5.	1	0	0	$= .05$
6.	0	2	1	$= .009375$

In Ex. 1. rejecting the 6 d. the half of the farthings is 1.5, and $12)1.5(125$; and this quot annexed to 528 completes the decimal, which is finite, there being no remainder.

In Ex. 2. the half of the farthings in 1 d. 1 f. is 2.5, and $12)2.5(2083$; which quot, annexed to 705, completes the decimal, being a mixt repeater.

In Ex. 3. rejecting 6 d. there remains 1 d. 3 f.; the half of which in farthings is 3.5, and $12)3.5(2916$; which quot, annexed to 982, completes the decimal, being another mixt repeater. The remaining three examples give finite decimals.

The reason of the above operation is obvious. For we reject every 6 d. because the farthings in 6 d. was converted to thousandth parts, by adding unity when the first three figures of the decimal were found: but the farthings in the other pence and farthings, being carried to the quot without any correction, are defective; and may be completed by saying, As 24 to 1, so the number of given farthings to the additional part; or, As 12 to 1, so half the number of farthings to the additional part; that is, we divide half the number of farthings by 12, and the quot annexed to the three figures first found, completes the decimal.

The decimals of pence and farthings may also be completed in the manner following.

Multiply by 4 the figures in the place of seconds and thirds, if less than 25; or multiply by 4, their excess, above 25, or above 50, or above 75; to the product add 1 for every 24 it contains; and you will have two places more to the decimal. If the fourth and fifth figures thus found be still imperfect, proceed, in like manner, to multiply them, or their excesses, by 4, to the product adding 1 for every 24 it contains, and you will gain two places more to the decimal; which will now either be finite, or will terminate in a repeating figure.

Thus, in Ex. 1. the excess of 28 above 25 is 3, and $3 \times 4 = 12$, the two subsequent figures of the decimal. But the decimal is still imperfect: for all decimals of

this sort are imperfect, except when the last two figures are 25, or 50, or 75, or when the decimal terminates in a repeating figure. Accordingly I proceed to find other two figures, saying, $12 \times 4 = 48$, and $48 + 2 = 50$, which completes the decimal.

In Ex. 2. I say, $05 \times 4 = 20$; but the decimal being still imperfect, I say, $20 \times 4 = 80$, and $80 + 3 = 83$. So the decimal repeats.

In Ex. 3. the excess of 82 above 75 is 7, and $7 \times 4 = 28$, and $28 + 1 = 29$. Again, $29 - 25 = 4$, and $4 \times 4 = 16$.

In Ex. 6. I say, $9 \times 4 = 36$, and $36 + 1 = 37$. Again, $37 - 25 = 12$, and $12 \times 4 = 48$, and $48 + 2 = 50$.

The reason of the rule now assigned will appear, by considering, that if 6 d. or 24 farthings, require the addition of an unit, to complete the decimal, every single farthing will require the addition of $\frac{1}{24}$ of an unit; but $\frac{1}{24} = .041\bar{6}$. If, therefore, the imperfect decimal for the farthings be multiplied by $.041\bar{6}$, the product will be the additional part. Thus, $.001$, the expression for one farthing, multiplied by $.041\bar{6}$, gives $.000041\bar{6}$ for the additional part.

$.001$

$.000041\bar{6}$

And $.001041\bar{6}$ is the decimal of 1 farthing complete.

But $.041\bar{6}$ being a troublesome multiplier, it is better to use the multiplier $.04 = \frac{1}{25}$; and as a twenty-fifth part is less than a twenty-fourth part, by 1 in 24, it is plain, that if you multiply by $.04$, or simply by 4, adding 1 for every 24 in the product, and annex the product thus increased to the imperfect expression for the farthings, you will have the same decimal as if you had multiplied by $.041\bar{6}$.

METHOD V.

The decimal for the parts of coin, weights, and measures, may easily and expeditiously be obtained, from
the

the following tables, constructed for that purpose: in using of which, take the decimals for the several given parts; add them, as taught in Addition of Decimals, and their sum is the decimal required.

EXAMPLE I.

Required the decimal of a pound, for 15 s. 7 $\frac{1}{2}$ d.

$$15 \text{ s.} = .75$$

$$7 \text{ d.} = .0291\bar{6}6666$$

$$\frac{1}{2} \text{ d.} = .00366458\bar{3}$$

$$\text{Ans. } .782831250 \text{ L.}$$

EXAMPLE II.

Required the decimal of one C. for 2 Q. 12 lb. 9 oz.

$$2 \text{ Q.} = .5$$

$$12 \text{ lb.} = .10714285,714285,$$

$$9 \text{ oz.} = .00502232,142857,$$

$$\text{Ans. } .61216517,857142, \text{ C.}$$

EXAMPLE III.

Required the decimal of a lb. Troy, for 10 oz. 7 dw. 1 gr.

$$10 \text{ oz.} = .83333333$$

$$7 \text{ dw.} = .0291\bar{6}666$$

$$1 \text{ gr.} = .001736\bar{1}$$

$$\text{Ans. } .8626736\bar{1} \text{ lb. Troy.}$$

The decimals in the tables are given complete, to answer such purposes as may require accuracy; but the practitioner may limit them to four, five, six, or any number of places, as he sees cause. The tables follow.

L. M O N E Y.

TABLE I. One pound the integer.

N.	Shillings.	pence.	farthings.	half-farthings.
1	.05	.0041 ϕ	.001041 ϕ	.0005208 z
2	.1	.008 z	.00208 z	.001041 ϕ
3	.15	.0125	.003125	.0015625
4	.2	.01 ϕ		.00208 z
5	.25	.0208 z		.0026041 ϕ
6	.3	.025		.003125
7	.35	.0291 ϕ		.0036458 z
8	.4	.0 z		
9	.45	.0375		
10	.5	.041 ϕ		
11	.55	.0458 z		
12	.6			
13	.05			
14	.7			
15	.7			
16	.8			
17	.85			
18	.9			
19	.95			

TABLE

TABLE II. One shilling the integer.

N.	Pence.	farthings.	half-farthings.
1	.08 $\frac{3}{4}$	.0208 $\frac{3}{4}$	.01041 $\frac{3}{8}$
2	.1 $\frac{6}{8}$	.041 $\frac{6}{8}$	.0208 $\frac{3}{4}$
3	.25	.0625	.03125
4	.3		.041 $\frac{6}{8}$
5	.41 $\frac{6}{8}$		.05208 $\frac{3}{4}$
6	.5		.0625
7	.58 $\frac{3}{4}$		.07291 $\frac{3}{8}$
8	. $\frac{6}{8}$		
9	.75		
10	.8 $\frac{3}{4}$		
11	.91 $\frac{6}{8}$		

2. AVOIRDUPOIS WEIGHT.

TABLE I. One tun the integer.

N.	C.	Q.	lb.	N.	C.	lb.
1	.05	.0125	.0004464,285714,	15	.75	.0066964,285714,
2	.1	.025	.0008928,571428,	16	.8	.0071428,571428,
3	.15	.0375	.0013392,857142,	17	.85	.0075892,857142,
4	.2		.0017857,142857,	18	.9	.0080357,142857,
5	.25		.0022321,428571,	19	.95	.0084821,428571,
6	.3		.0026785,714285,	20		.0089285,714285,
7	.35		.003125	21		.009375
8	.4		.0035714,285714,	22		.0098214,285714,
9	.45		.0040178,571428,	23		.0102678,571428,
10	.5		.0044642,857142,	24		.0107142,857142,
11	.55		.0049107,142857,	25		.0111607,142857,
12	.6		.0053571,428571,	26		.0116071,428571,
13	.65		.0058035,714285,	27		.0120535,714285,
14	.7		.00625			

TABLE

TABLE II. One C. or 112 lb. the integer.

N	Q.	lb.	oz.	N	lb.
1	.25	.00892857, 142857,	.00055803, 571428,	16	.14285714, 285714,
2	.5	.01785714, 285714,	.00111607, 142857,	17	.15178571, 428571,
3	.75	.02678571, 428571,	.00167410, 714285,	18	.16071428, 571428,
4		.03571428, 571428,	.00223214, 285714,	19	.16964285, 714285,
5		.04464285, 714285,	.00279017, 857142,	20	.17857142, 857142,
6		.05357142, 857142,	.00334821, 428571,	21	.1875
7		.0625	.00390625	22	.19642857, 142857,
8		.07142857, 142857,	.00446428, 571428,	23	.20535714, 285714,
9		.08035714, 571428,	.00502232, 142857,	24	.21428571, 428571,
10		.08928571, 428571,	.00558035, 714285,	25	.22321428, 571428,
11		.09821428, 571428,	.00613839, 285714,	26	.23214285, 714285,
12		.10714285, 714285,	.00669642, 857142,	27	.24107142, 857142,
13		.11607142, 857142,	.00725446, 428571,		
14		.125	.0078125		
15		.13392857, 142857,	.00837053, 571428,		

TABLE III. One lb. the integer.

N	oz.	drams.	N	oz.	drams.	N	oz.	drams.
1	.0625	.00390625	6	.375	.0234375	11	.6875	.04296875
2	.125	.0078125	7	.4375	.02734375	12	.75	.046875
3	.1875	.01171875	8	.5	.03125	13	.8125	.05078125
4	.25	.015625	9	.5625	.03515625	14	.875	.0546875
5	.3125	.01953125	10	.625	.0390625	15	.9375	.05859375

3. TROY WEIGHT.

TABLE I. One lb. the integer.

N	oz.	dwt.	grains.	N	dwt.	grains.
1	.083	.00416	.00017361	13	.05416	.00225694
2	.16	.0083	.0003472	14	.0583	.0024308
3	.25	.0125	.00052083	15	.0625	.00260416
4	.33	.016	.000694	16	.06	.0027
5	.416	.02083	.00086803	17	.07083	.00295138
6	.5	.025	.0010416	18	.075	.003125
7	.583	.02916	.00121521	19	.07916	.00329861
8	.66	.033	.001388	20		.003472
9	.75	.0375	.0015625	21		.00364583
10	.83	.0416	.0017361	22		.0038194
11	.916	.04583	.00190972	23		.00399308
12		.05	.002083			

TABLE

TABLE II. One ounce the integer.

N.	dw.	grains.	N.	dw.	grains.
1	.05	.002083	13	.65	.027083
2	.1	.00416	14	.7	.02916
3	.15	.00625	15	.75	.03125
4	.2	.0083	16	.8	.03
5	.25	.010416	17	.85	.035416
6	.3	.0125	18	.9	.0375
7	.35	.014583	19	.95	.039583
8	.4	.016	20		.0416
9	.45	.01875	21		.04375
10	.5	.02083	22		.04583
11	.55	.022916	23		.047916
12	.6	.025			

4. APOTHECARIES WEIGHT.

TABLE. One lb. the integer.

N.	oz.	drams.	scruples.	grains.	N.	grains.
1	.083	.010416	.003472	.00017361	12	.002083
2	.16	.02083	.006944	.00034722	13	.00225694
3	.25	.03125		.00052083	14	.0024308
4	.33	.0416		.00069444	15	.00260416
5	.416	.052083		.00086808	16	.00277778
6	.5	.0625		.00104167	17	.00295138
7	.583	.072916		.00121528	18	.003125
8	.66			.00138889	19	.00329861
9	.75			.0015625		
10	.83			.00173611		
11	.916			.00190972		

5. WOOL:

5. WOOL-WEIGHT.

TABLE I. One last the integer.

<i>N.</i>	<i>Sacks.</i>	<i>stones.</i>	<i>lb.</i>	<i>N.</i>	<i>stones.</i>
1	.08 $\frac{3}{4}$	.0032,051282,	.0002,289377,	14	.0448,717948,
2	.1 $\frac{1}{2}$	.0064,102564,	.0004,578754,	15	.0480,769230,
3	.25	.0096,153846,	.0006,868131,	16	.0512,820512,
4	.3	.0128,205128,	.0009,157509,	17	.0544,871794,
5	.41 $\frac{1}{2}$	.0160,256410,	.0011,446886,	18	.0576,923076,
6	.5	.0192,307692,	.0013,736263,	19	.0608,974358,
7	.58 $\frac{3}{4}$	.0224,358974,	.0016,025641,	20	.0641,025641,
8	. $\frac{3}{4}$	.0256,410256,	.0018,315018,	21	.0673,076923,
9	.75	.0288,461538,	.0020,604395,	22	.0705,128205,
10	.8 $\frac{3}{4}$	.0320,512820,	.0022,893772,	23	.0737,179487,
11	.91 $\frac{1}{2}$	.0352,564102,	.0025,183150,	24	.0769,230769,
12		.0384,615384,	.0027,472527,	25	.0801,282051,
13		.041 $\frac{1}{2}$	.0029,761904,		

TABLE II. One sack the integer.

<i>N.</i>	<i>Stones.</i>	<i>lb.</i>	<i>N.</i>	<i>stones.</i>
1	.03,846153,	.00,274725,	14	.53,846153,
2	.07,692307,	.00,549450,	15	.57,692307,
3	.11,538461,	.00,824175,	16	.61,538461,
4	.15,384615,	.01,098901,	17	.65,384615,
5	.19,230769,	.01,373626,	18	.69,230769,
6	.23,076923,	.01,648351,	19	.73,076923,
7	.26,923076,	.01,923076,	20	.76,923076,
8	.30,769230,	.02,197802,	21	.80,769230,
9	.34,615384,	.02,472527,	22	.84,615384,
10	.38,461538,	.02,747252,	23	.88,461538,
11	.42,307692,	.03,021978,	24	.92,307692,
12	.46,153846,	.03,296703,	25	.96,153846,
13	.5	.03,571428,		

6. DRY

6. DRY MEASURE.

TABLE I. One load the integer.				TABLE II. One quarter the integer.		TABLE III. One bushel the integer.	
N.	Q.	Buf.	Gallons.	Buf.	Gallons.	Gall.	Pints.
1	.2	.025	.003125	.125	.015625	.125	.015625
2	.4	.05	.00625	.25	.03125	.25	.03125
3	.6	.075	.009375	.375	.046875	.375	.046875
4	.8	.1	.0125	.5	.0625	.5	.0625
5		.125	.015625	.625	.078125	.625	.078125
6		.15	.01875	.75	.09375	.75	.09375
7		.175	.021875	.875	.109375	.875	.109375

7. WINE-MEASURE.

TABLE I. One tun the integer.

N.	Hbds	Gallons.	N.	Gallons.	N.	Gallons.
1	.25	.00,396825	22	.08,730158,	43	.17,003492,
2	.5	.00,793650,	23	.09,126984,	44	.17,460317,
3	.75	.01,190476,	24	.09,523809,	45	.17,857142,
4		.01,587301,	25	.09,920634,	46	.18,253908,
5		.01,984126,	26	.10,317460,	47	.18,650793,
6		.02,380952,	27	.10,714285,	48	.19,047619,
7		.02,777777,	28	.11,111111,	49	.19,444444,
8		.03,174603,	29	.11,507936,	50	.19,841209,
9		.03,571428,	30	.11,904761,	51	.20,238095,
10		.03,968253,	31	.12,301587,	52	.20,634920,
11		.04,365079,	32	.12,698412,	53	.21,031746,
12		.04,761904,	33	.13,095238,	54	.21,428571,
13		.05,158730,	34	.13,492063,	55	.21,825396,
14		.05,555555,	35	.13,888888,	56	.22,222222,
15		.05,952380,	36	.14,285714,	57	.22,619047,
16		.06,349206,	37	.14,682539,	58	.23,015873,
17		.06,746031,	38	.15,079365,	59	.23,412698,
18		.07,142857,	39	.15,476190,	60	.23,809523,
19		.07,539682,	40	.15,873015,	61	.24,206349,
20		.07,936507,	41	.16,269841,	62	.24,603174,
21		.08,333333,	42	.16,666666,		

TABLE II. One hoghead the integer.

N.	Gallons.	N	Gallons.	Pints.
60	.952,380952,	1	.015,873015,	.001,984126,
50	.793,650793,	2	.031,746031,	.003,968253,
40	.634,920634,	3	.047,619047,	.005,952380,
30	.476,190476,	4	.063,492063,	.007,936507,
20	.317,460317,	5	.079,365079,	.009,920634,
10	.158,730158,	6	.095,238095,	.011,904761,
		7	.111,111111,	.013,888888,
		8	.126,984126,	
		9	.142,857142,	

8. BEER-MEASURE.

TABLE I.				TABLE II.		
One tun the integer.				One hoghead the integer.		
N.	Hbds	Firkins.	Gallons.	Fir.	Gallons.	Pints.
1	.25	.0416	.004,629,	.16	.0185,185,	.0023,148,
2	.5	.083	.009,259,	.32	.0370,370,	.0046,296,
3	.75	.125	.013,888,	.5	.0555,555,	.0069,444,
4		.16	.018,518,	.67	.0740,740,	.0092,592,
5		.2083	.023,148,	.83	.0925,925,	.0115,740,
6			.027,777,		.1111,111,	.0138,888,
7			.032,407,		.1296,296,	.0162,037,
8			.037,037,		.1481,481,	

TABLE

TABLE III. One hoghead of 51 gallons, 8 pints each, the integer.

N.	Gallons.	N.	Pints.
50	.980,3921568627450980,	1	.002,4509803921568627,
40	.784,3137254901960784,	2	.004,9019607843137254,
30	.588,2352941176470588,	3	.007,3529411764705882,
20	.392,1568627450980392,	4	.009,8039215686274509,
10	.196,0784313725490196,	5	.012,2549019607843137,
9	.176,4705882352941176,	6	.014,7058823529411764,
8	.156,8627450980392156,	7	.017,1568627450980392,
7	.137,2559019607843137,		
6	.117,6470588235294117,		
5	.098,0392156862745098,		
4	.078,4313725490196078,		
3	.058,8235294117647058,		
2	.039,2156862745098039,		
1	.019,6078431372549019,		

TABLE IV. One Scots hoghead the integer.

N.	Gall.	pints.	N.	Gall.	pints.	N.	Gall.
1	.0625	.0078125	6	.375	.046875	11	.6875
2	.125	.015625	7	.4375	.0546875	12	.75
3	.1875	.0234375	8	.5		13	.8125
4	.25	.03125	9	.5625		14	.875
5	.3125	.0390625	10	.625		15	.9375

9. CLOTH-MEASURE.

TABLE I.			TABLE II.			TABLE III.		
One yard the integer.			One ell Flemish the integer.			One ell English the integer.		
N.	Q.	Nails.	N.	Q.	Nails.	N.	Q.	Nails.
1	.25	.0625	1	.3	.083	1	.2	.05
2	.5	.125	2	.6	.16	2	.4	.1
3	.75	.1875	3		.243	3	.6	.15
						4	.8	

10. LONG MEASURE.

TABLE I. One mile the integer.

N.	Furl.	chains.	poles.	links.	N.	Links.	N.	Links.
1	.125	.0125	.003125	.000125	10	.00125	19	.002375
2	.25	.025	.00625	.00025	11	.001375	20	.0025
3	.375	.0375	.009375	.000375	12	.0015	21	.002625
4	.5	.05		.0005	13	.001625	22	.00275
5	.625	.0625		.000625	14	.00175	23	.002875
6	.75	.075		.00075	15	.001875	24	.003
7	.875	.0875		.000875	16	.002		
8		.1		.001	17	.002125		
9		.1125		.001125	18	.00225		

TAB. II. One chain the integer.

N.	pol.	lin.	N.	lin.	N.	lin.
1	.25	.01	10	.10	19	.19
2	.5	.02	11	.11	20	.20
3	.75	.03	12	.12	21	.21
4		.04	13	.13	22	.22
5		.05	14	.14	23	.23
6		.06	15	.15	24	.24
7		.07	16	.16		
8		.08	17	.17		
9		.09	18	.18		

TABLE III. One pole the integer.

N.	Yards.	feet.	inches.	N.	inches.
1	.1,81,	.0,60,	.0,05,	10	.0,50,
2	.3,63,	.1,21,	.0,10,	11	.0,55,
3	.5,45,		.0,15,		
4	.7,27,		.0,20,		
5	.9,09,		.0,25,		
6			.0,30,		
7			.0,35,		
8			.0,40,		
9			.0,45,		

TABLE IV. One yard the integer.

N.	Feet	inches.
1	.3	.027
2	.6	.05
3		.083
4		.1
5		.138
6		.16
7		.194
8		.2
9		.25
10		.27
11		.308

TABLE V. One foot the integer.

N.	Inch.	lines.
1	.083	.00694
2	.16	.0138
3	.25	.02083
4	.3	.027
5	.416	.03477
6	.5	.0416
7	.583	.04861
8	.6	.05
9	.75	.0625
10	.83	.0694
11	.916	.07638

TABLE

TABLE VI. One Scots mile the integer.

<i>N</i>	<i>Furl.</i>	<i>chains.</i>	<i>falls.</i>	<i>ells.</i>
1	.125	.0125	.003125	.00052083
2	.25	.025	.00625	.0010416
3	.375	.0375	.009375	.00156249
4	.5	.05		.00208333
5	.625	.0625		.00260416
6	.75	.075		
7	.875	.0875		
8		.1		
9		.1125		

II. LAND-MEASURE.

TABLE I. One acre the integer.

<i>N.</i>	<i>Rds.</i>	<i>sq. poles.</i>	<i>sq. links.</i>	<i>N.</i>	<i>sq. link.</i>
1	.25	.00625	.00001	40	.0004
2	.5	.0125	.00002	50	.0005
3	.75	.01875	.00003	60	.0006
4		.025	.00004	70	.0007
5		.03125	.00005	80	.0008
6		.0375	.00006	90	.0009
7		.04375	.00007	100	.001
8		.05	.00008	200	.002
9		.05625	.00009	300	.003
10		.0625	.0001	400	.004
20		.125	.0002	500	.005
30		.1875	.0003	600	.006

TABLE

TABLE II. One Scots acre the integer.

N.	Rds	<i>sq. falls.</i>	<i>sq. ells.</i>
1	.25	.00625	.0001736 $\frac{1}{2}$
2	.5	.0125	.000347 $\frac{1}{2}$
3	.75	.01875	.0005208 $\frac{3}{4}$
4		.025	.000694
5		.03125	.0008680 $\frac{1}{2}$
6		.0375	.001041 $\frac{1}{2}$
7		.04375	.0012152 $\frac{1}{2}$
8		.05	.00138
9		.05625	.0015625
10		.0625	.001736 $\frac{1}{2}$
20		.125	.00347 $\frac{1}{2}$
30		.1875	.005208 $\frac{3}{4}$

12. SUPERFICIAL MEASURE.

TABLE I. One square yard the integer.

N.	<i>Sq. ft</i>	<i>square inches.</i>	N.	<i>Square inches.</i>
1	.1	.0007,716049382,	10	.0077,160493827,
2	.2	.0015,432098765,	20	.0154,329987654,
3	.3	.0023,148148148,	30	.0231,481481481,
4	.4	.0030,864197530,	40	.0308,641975308,
5	.5	.0038,580246913,	50	.0385,802469135,
6	.6	.0046,296296296,	60	.0462,962962962,
7	.7	.0054,012345679,	70	.0540,123456790,
8	.8	.0061,728395061,	80	.0617,283950611,
9	.9	.0069,444444444,	90	.0694,444444444,
			100	.0771,604938271,

TABLE

TABLE II. One foot the integer.

N.	sq. inch.	N.	sq. inch.	N.	sq. inch.	N.	sq. inch.
1	.00694	6	.0416	20	.138	70	.486x
2	.0138	7	.0486x	30	.208x	80	.x
3	.0208x	8	.0x	40	.27	90	.625
4	.027	9	.0625	50	.347x	100	.694
5	.0347x	10	.0694	60	.416		

13. SOLID MEASURE.

TABLE I. One cubic yard the integer.

N.	Cu. F.	N.	Cu. F.	N.	Cu. F.	N.	Cu. F.	N.	Cu. F.
1	.037,	7	.259,	13	.481,	19	.703,	25	.925,
2	.074,	8	.296,	14	.518,	20	.740,	26	.962,
3	.111,	9	.333,	15	.555,	21	.777,		
4	.148,	10	.370,	16	.592,	22	.814,		
5	.185,	11	.407,	17	.629,	23	.851,		
6	.222,	12	.444,	18	.666,	24	.888,		

TABLE II. One cubic foot the integer.

N.	Cubic Inches.	N.	Cubic Inches.	N.	Cubic Inches.
1	.000578,703,	10	.005787,037,	100	.057870,370,
2	.001157,407,	20	.011574,074,	200	.115740,740,
3	.001736,111,	30	.017361,111,	300	.173611,111,
4	.002314,814,	40	.023148,148,	400	.231481,481,
5	.002893,518,	50	.028935,185,	500	.289351,851,
6	.003472,222,	60	.034722,222,	600	.347222,222,
7	.004050,925,	70	.040509,259,	700	.405091,592,
8	.004629,629,	80	.046296,296,	800	.462962,962,
9	.005208,333,	90	.052083,333,	900	.520833,333,
				1000	.578703,703,

14. A CIRCLE.

TABLE I. One circle the integer.

<i>N.</i>	<i>Sig.</i>	<i>degr.</i>	<i>min.</i>
1	.083	.0027	.00004,629,
2	.16	.005	.00009,259,
3	.25	.0083	.00013,888,
4	.3	.01	.00018,518,
5	.416	.0138	.00023,148,
6	.5	.016	.00027,777,
7	.583	.0194	.00032,407,
8	.6	.02	.00037,037,
9	.75	.025	.00041,666,
10	.83	.027	.00046,296,
11	.916		
20		.03	.00092,592,
30			.00138,888,
40			.00185,185,
50			.00231,481,

TABLE II. One sign the integer.

<i>N.</i>	<i>Deg.</i>	<i>min.</i>	<i>sec.</i>
1	.03	.0003	.00000,925,
2	.06	.001	.00001,851,
3	.1	.0016	.00002,777,
4	.13	.002	.00003,703,
5	.16	.0027	.00004,629,
6	.2	.003	.00005,555,
7	.23	.0038	.00006,481,
8	.26	.004	.00007,407,
9	.3	.005	.00008,333,
10	.3	.005	.00009,259,
20	.6	.01	.00018,518,
30		.016	.00027,777,
40		.02	.00037,037,
50		.027	.00046,296,

15. TIME.

TABLE I. One year of 365 days the integer.

N.	Days.	N.	Days.	N.	Days.
1	.00,27397260,	8	.02,19178082,	60	.16,43835616,
2	.00,54794520,	9	.02,46575342,	70	.19,17808219,
3	.00,82191780,	10	.02,73972602,	80	.21,91780821,
4	.01,09589041,	20	.05,47945205,	90	.24,65753424,
5	.01,36986301,	30	.08,21917808,	100	.27,39726027,
6	.01,64383561,	40	.10,95890410,	200	.54,79452054,
7	.01,91780821,	50	.13,69863013,	300	.82,19178082,

TABLE II. One year of 12 months, 28 days each, the integer.

N.	Months.	weeks.	days.	N.	Months.
1	.07,692307,	.01,923076,	.00,274725,	7	.53,846153,
2	.15,384615,	.03,846153,	.00,549450,	8	.61,538461,
3	.23,076923,	.05,769230,	.00,824175,	9	.69,230769,
4	.30,769230,		.01,098901,	10	.76,923076,
5	.38,461538,		.01,373626,	11	.84,615384,
6	.46,153846,		.01,648351,	12	.92,307692,

TAB. III. One month of 28 days the integer.

N.	W.	days.
1	.25	.03,571428,
2	.5	.07,142857,
3	.75	.10,714285,
4		.14,285714,
5		.17,857142,
6		.21,428571,

TAB. IV. One day the integer.

N.	Hour.	minutes.	N.	Minut.
1	.0416	.000694	30	.02083
2	.083	.00138	40	.027
3	.125	.002083	50	.03472
4	.16	.0027		
5	.2083	.003472		
6	.25	.00416		
7	.2916	.004861		
8	.33	.005556		
9	.375	.00625		
10	.416	.00694		
20	.83	.0138		

TABLE V. One hour, or one degree, the integer.

N.	Min.	sec.	N.	Min.	sec.
1	.016	.00027	10	.16	.0027
2	.03	.0005	20	.3	.005
3	.05	.00083	30	.5	.0083
4	.06	.001	40	.6	.01
5	.083	.00138	50	.83	.0138
6	.1	.0016			
7	.116	.00194			
8	.13	.002			
9	.15	.0025			

P R O B. III.

To reduce the remainder of a division to a decimal.

R U L E.

The remainder being the numerator, and the divisor the denominator of a vulgar fraction, after placing the decimal point on the right of the integral part of the quot, annex ciphers to the remainder; then continue the division till o remain, or till the quot repeat or circulate, or till you think proper to limit the decimal; and the number on the right of the point is a decimal of the integer expressed in the quot.

Example 1.

Divide 513 l. among 36 men

$$\begin{array}{r}
 \text{L.} \\
 36 \overline{) 513} (14.25 \\
 \underline{36} \\
 153 \\
 \underline{144} \\
 \text{Rem. } 90 \\
 \underline{72} \\
 180 \\
 \underline{180} \\
 (0)
 \end{array}$$

Example 2.

Divide 176 s. among 24 boys.

$$\begin{array}{r}
 \text{s.} \\
 24 \overline{) 176} (7.3 \\
 \underline{168} \\
 \text{Rem. } 80 \\
 \underline{72} \\
 (8)
 \end{array}$$

Ex-

Example 3.

Divide 267 guineas among 33 failors.

$$\begin{array}{r} \text{g.} \\ 33 \overline{) 267} (8.09, \\ \underline{264} \\ \text{Rem. } * 300 \\ \underline{297} \\ * 3021 \end{array}$$

Example 4.

Divide 409 C. into 81 equal parcels. C.

$$\begin{array}{r} 81 \overline{) 409} (5.04938 + \\ \underline{405} \\ \text{Rem. } 400 \\ \underline{324} \\ 760 \\ \underline{729} \\ 310 \\ \underline{243} \\ 670 \\ \underline{648} \\ (22) \end{array}$$

P R O B. IV.

To reduce a decimal to value.

R U L E.

Multiply the given decimal by the number of parts of the next inferior denomination contained in an unit of the integer; and from the product point off so many figures to the right hand as there are places in the given decimal. On the left hand of the point are parts, and on the right a decimal of one of these parts; which decimal must be reduced in the same manner to the next inferior denomination, and from that to the next, and so on to the lowest; the several figures on the left of the points are parts; and if there be still some figure or figures on the right, they are a decimal of the lowest of the parts.

G 2

Ex-

Example 1.

Reduce .875 l. to value.

$$\begin{array}{r} \text{L.} \quad \text{s.} \quad \text{d.} \\ .875 = 17 \quad 6 \\ \underline{20} \end{array}$$

$$\text{s. } 17.500$$

$$\underline{12}$$

$$\text{d. } 6.0$$

Example 2.

Reduce .769 l. to value.

$$\begin{array}{r} \text{L.} \quad \text{s.} \quad \text{d.} \quad \text{f.} \\ .769 = 15 \quad 4 \quad 2 \\ \underline{20} \end{array}$$

$$\text{s. } 15.3820$$

$$\underline{12}$$

$$\text{d. } 4.584$$

$$\underline{4}$$

$$\text{f. } 2.336$$

Example 3.

Reduce .9375 C. to value.

$$\begin{array}{r} \text{C.} \quad \text{Q.} \quad \text{lb.} \\ .9375 = 3 \quad 21 \\ \underline{4} \end{array}$$

$$\text{Q. } 3.7500$$

$$\underline{28}$$

$$6.00$$

$$\underline{15.00}$$

$$\text{lb. } 21.00$$

Example 4.

Reduce .84362 lb. Troy to value.

$$\begin{array}{r} \text{lb. Troy.} \quad \text{oz.} \quad \text{dw.} \quad \text{g.} \\ .84362 = 10 \quad 2 \quad 11 \\ \underline{12} \end{array}$$

$$\text{oz. } 10.12344$$

$$\underline{20}$$

$$\text{dw. } 2.46880$$

$$\underline{24}$$

$$18752$$

$$\underline{9376}$$

$$\text{gr. } 11.2512$$

This problem is the converse of Problem II. Method II. and the one operation serves as a proof of the other. The reason of pointing the product, as the rule directs, is plain. For, in Ex. 1. as $1000 : 875 :: 20 : 17$; that is, as the decimal denominator to the decimal numerator, so the vulgar denominator to the vulgar numerator.

In Ex. 1. and 3. the full value of the decimal comes out in parts, the decimal being quite exhausted; but in

Ex.

Ex. 2. besides the parts, there is a decimal of a farthing, viz. .336 f.; and in Ex. 4. besides the parts, we have a decimal of a grain, viz. .2512 gr.

MORE EXAMPLES.

5. .876 L. = 17 s. 6d. 0.96 f.
6. .74 guinea = 15 s. 6d. 1.92 f.
7. .43569 C. = 1 Q. 20 lb. 12.75648 oz.
8. .91112 lb. Troy = 10 oz. 18 dw. 16.0512 gr.
9. .95 yard = 2 feet 10.2 inches.
10. .746 acre = 2 roods 39 falls 12.96 ells Scots.

If the given decimal be a repeater, pure or mixt, in order to obtain the complete value, carry at 9 on the right hand; and if the multiplier have a cipher on the right, instead of annexing the cipher, annex the right-hand figure of the product. If the multiplier repeat, work as taught in multiplication of repeating decimals.

Ex. 1.	Ex. 2.	Ex. 3.
L. s. d.	L. s. d.	Mark. s. d. f.
.6 = 13 4	.93 = 18 8	3).4453125 = 5 11 1
20	20	13.3
s. 13.3	s. 18.66	13359375
12	12	4453125
d. 4.0	d. 8.0	5.7890625
		1484375
		s. 5.9375000
		12
		d. 11.2500
		4
		f. 1.00

Ex.

Ex. 4.

C. Q. lb.
 $.3 = 1 \quad 9\frac{1}{3}$

4

Q. 1.3
 28

 28
 6

 lb. 9.3

Ex. 5.

lb. Troy. oz. dw.
 $.62083 = 7 \quad 9$

12

oz. 7.45000
 20

 dw. 9.00

If the given decimal be a circulate, in order to obtain the value accurately, we may work as taught in addition and multiplication of circulates.

Ex. 1.

C. Q. lb.
 $.6696, 428571, = 2 \quad 19$

4

Q. 2.6785, 714285,
 28

5 4285, 714285,

13 5714, 285714,

lb. 19.0000000000

Ex. 2.

Pole. yd. ft. in.
 $.87, = 4 \quad 2 \quad 6$

5.5

4,39

43,93

yds. 4.833

3

feet. 2.500

12

inch. 6.0

But such operations being commonly tedious, it is more usual to take part of the circulate as an approximate, and find the value nearly by the general rule explained above.

The decimal of a pound Sterling may be reduced to value by inspection, in the following manner.

Double the figure in the place of primes for shillings; and if the figure in the place of seconds be 5, or exceed 5, reckon 1 shilling more; and rejecting 5 in the second place,

place, the figures in the second and third places are so many farthings, abating 1 for every 25.

E X A M P L E S.

	L.	s.	d.	f.
1.	.718	= 14	4	2
2.	.759	= 15	2	1
3.	.894	= 17	10	3
4.	.92	= 18	5	0
5.	.68	= 13	7	1
6.	.072	= 1	5	2
7.	.9276	= 18	6	2
8.	.87638	= 17	6	1

In Example 1. the figure 7 doubled gives 14 s.; the two following figures 18 are farthings, equal to 4 d. 2 f.

In Example 2. the figure 7 doubled gives 14 s. and 5 in the place of seconds gives 1 shilling more, in all 15 s.; and the other figure 9 is farthings, viz. 2 d. 1 f.

In Example 3. the figure 8 in the place of primes, and 5 in the place of seconds, give 17 s.; the remaining figures 44, abating 1, are farthings, viz. 10 d. 3 f.

In Example 4. and 5. the place of thirds must be supposed to be filled with a cipher; and so, in Ex. 4. we have 20 f. = 5 d.; and in Ex. 5. we have 30 f. — 1 = 7 d. 1 f.

In Example 6. the place of primes affords no shillings; but we have 1 s. from the place of seconds.

In Example 7. and 8. the figures to the right of the place of thirds, are rejected, as being in value less than a farthing.

When the figures in the second and third place to be converted into farthings are 25, the answer, by inspection, comes out exact, viz. 24 f. or 6 d.; but in all other cases, the answer, by inspection, is too great, no allowance or correction being made till the convertible number amount to 25, and afford a deduction of 1 farthing complete. Hence, by inspection, we have frequently

quently 1 farthing more than by the common method; but the two methods will agree, or give the same answer, if, from the figures to be turned into farthings, we subtract their 25th part, esteeming the remainder farthings and decimal parts of a farthing.

Thus, $.718 \text{ l.} = 14 \text{ s. } 4 \text{ d. } 2 \text{ f.}$ by inspection; but by the common method, and by inspection corrected, the answer comes out 1 farthing less, as follows.

Common method.

$$\begin{array}{r} \text{L.} \\ .718 \\ \underline{20} \\ \text{s. } 14.360 \\ \underline{12} \\ \text{d. } 4.32 \\ \underline{4} \\ \text{f. } 11.28 \end{array}$$

Inspection corrected.

$$\begin{array}{r} \text{If } 25 : 1 :: 18 : .72. \\ \text{that is, } 25 \overline{) 18.0} (.72 \\ \underline{17 \ 5} \\ 50 \text{ and } 18 \\ \underline{50} \\ (0) \quad \underline{.72} \\ 17.28 \end{array}$$

d. f.

And $17.28 = 4 \text{ } 1.28$

To conclude, instead of dividing by 25, we may multiply by .04; and then the exact value of any decimal of a pound Sterling may be found as follows.

From the primes and seconds set off the shillings; multiply the remainder by 4, setting the product two places to the right; subtract the product from the first remainder; and from the second remainder point off so many places to the right as there are figures in the first remainder. The number on the left of the point is farthings, and the figures on the right are a decimal of a farthing.

Example 1.

$$\begin{array}{r} \text{s. d. f.} \\ .718 \text{ l.} = 14 \ 4 \ 1.28 \\ \hline 1. \text{ Rem. } .18 \\ 72 = 18 \times 4 \\ \hline 2. \text{ Rem. } 17.28 \end{array}$$

Example 2.

$$\begin{array}{r} \text{s. d. f.} \\ .769 \text{ l.} = 15 \ 4 \ 2.336 \\ \hline 1. \text{ Rem. } .191 \\ 764 = 191 \times 4 \\ \hline 2. \text{ Rem. } 18.336 \end{array}$$

The

The value of a decimal is easily and readily found from the following tables, calculated for that purpose.

The tables are constructed by finding the value of unity in the place of primes, seconds, thirds, fourths, &c.; that is, by finding the value of .1, of .01, of .001, .0001, &c. and placing these values under the top-figures 1, 2, 3, 4, &c. The values of 2 in the place of primes, seconds, thirds, &c. are found by doubling the values of unity already found; the values of 3 are found by adding the values of 2 to the values of 1; the values of 4 are found by adding the values of 3 to the values of 1; and in like manner are found the values of the other digits.

In using the tables, take the value of the figure in the place of primes from under the top-figure 1; take the value of the figure in the place of seconds from under the top-figure 2, &c. The sum of these values is the total value of the given decimal.

I. M O N E Y.

E X A M P L E I.

Required the value of .775 l. Sterling.

	s.	d.	f.
.7 =	14		
.07 =	1	4	3.2
.005 =		1	0.8

Total value 15 6

The value of the first and second figures of the given decimal may generally with ease be taken out at once: thus,

	s.	d.	f.
.77 =	15	4	3.2
.005 =		1	0.8

Total value 15 6

EXAMPLE II.

Required the value of .795 l. Sterling.

	s.	d.	f.
.79 =	15	9	2.4
.005 =		1	0.8

Total value 15 10 3.2

If the given decimal consist of more places than there are top-figures in the table, in this case the values under the top-figure 5 are found by moving the decimal points under the top-figure 4, one place more to the left; and the values under the top-figure 6 are found by moving the decimal points under 4, two places more to the left, &c.

EXAMPLE III.

Required the value of .634375 l. Sterling.

	s.	d.	f.
.63 =	12	7	0.8
.004 =			3.84
.0003 =			.288
.00007 =			.0672
.000005 =			.0048

Total value 12 8 1.0000

If the given decimal be a repeater, pure or mixt, take successive values of the repeating figure, till the columns of the decimals begin to reiterate; that is, till the column of primes be reiterated in the column of seconds, or till the column of seconds be reiterated in the column of thirds, &c. Begin the addition at the column on the left of the first reiteration; in adding this column carry at 9, and you have the value complete.

EX-

EXAMPLE IV.

Required the value of $.3$ l. Sterling.

	s.	d.	f.
$.33 =$	6	7	0.8
$.003 =$			2.88
$.0003 =$			.288
$.00003 =$			.0288
Value	6	8	0.0 complete.

In the above example I take successive values of the repeating figure 3 till I perceive that the column of primes is reiterated in the column of seconds; then I add the column of primes, as being the column on the left of the first reiteration; I carry at 9, and find the complete value of the given decimal to be 6s. 8d. The columns on the right of the primes are rejected, as being, when completed by assuming more successive values of 3 , so many pure repetitions of the column of primes.

EXAMPLE V.

Required the value of $.916$ l. Sterling.

	s.	d.	f.
$.91 =$	18	2	1.6
$.006 =$		1	1.76
$.0006 =$			.576
$.00006 =$			.0576
Value	18	4	0.0 complete.

EXAMPLE VI.

Required the value of $.9989583$ l. Sterling.

	s.	d.	f.
.99	= 19	9	2.4
.008	=	1	3.68
.0009	=		.864
.00005	=		.0480
.000008	=		.00768
.0000003	=		.000288
.00000003	=		.0000288
.000000003	=		.00000288

Value 19 11 3.00000 complete.

EXAMPLE VII.

Required the value of .86 $\frac{1}{2}$ l. Sterling.

	s.	d.	f.
.86	= 17	2	1.6
.001	=		.96
.0001	=		.096

Value 17 2 2.6 complete.

EXAMPLE VIII.

Required the value of .84 $\frac{1}{2}$ l. Sterling.

	s.	d.	f.
.84	= 16	9	2.4
.00 $\frac{1}{2}$	=	1	2.72
.0007	=		.672
.00007	=		.0672

Value 16 11 1.86 complete.

EXAMPLE IX.

Required the value of .45 $\frac{1}{2}$ s.

	<i>d.</i>	<i>f.</i>
.4	= 4	3.2
.05	=	2.4
.008	=	.384
.0008	=	.0384
.00008	=	.00384

Value 5 2.026 complete.

2. AVOIRDupois-WEIGHT.

When the given decimal is a circulate, as often happens in Avoirdupois-weight, wool-weight, wine and beer measure, long measure of one pole the integer, superficial and solid measure, a circle, and time; in this case, the exact value of the given decimal may be found by completing the circles of the separate values of the figures that constitute the circle of the given decimal, and adding as directed in addition of circulates. Here observe, that the said circles may always be completed from the decimals under the top-figure that denotes the number of places in the circle of the given decimal, or from the decimals under any top-figure that is higher than it, and frequently by a simple reiteration of the figures in the finite part.

EXAMPLE I.

Required the value of .07, 142857, C.

	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
.07	= 7	13	7.04
.001	=	1	12.672
.0004	=		11.4688
.00002	=		.57344
.000008	=		.229376
.0000005	=		.143360
.00000007	=		.200704

Thus it stands as taken from the tables; but after separating the circles from the finite parts, and completing the circles from the decimals under the top-figure

gure 6, and adding all into one sum, it will stand as follows.

lb.	oz.	dr.
7	13	7.04,
	I	12.67,202867,
		11.46,881146,
		.57,344057,
		.22,937622,
		.1,433601,
		.200704,
7	15	15.99,999999,

Value 8 0 0.00 complete.

The sum of the circles being a series of nines, their value is unity; which being accordingly added to the finite part, completes the value of the given decimal.

Though most of the other tables have only four top-figures, yet the circles of the several figures that constitute the circle of the given decimal may be found, by moving the decimal points under the top-figure 4 toward the left, till you have the value of the several figures under the top-figure that denotes the number of places in the circle of the given decimal; and then the several circles may be completed the same way, as in the example above. For further illustration take the following example of wine-measure.

EXAMPLE II.

Required the value of .81,349206, tun.

.8

	<i>Hbds.</i>	<i>g.</i>	<i>p.</i>
.8	= 3	12	4.80,
.01	=	2	4.16,
.003	=		6.04,800604,
.0004	=		.80,640080,
.00009	=		.18,144018,
.000002	=		.403200,
.00000006	=		.12096,
	3	15	7.99,999999,
			1,

Value 3 16 0 complete.

But few cases require such accuracy. The value of a decimal may generally be found to sufficient exactness from the tables, by taking the separate values of the first four, or at most of the first six figures, and using two decimal places only. Example I. done in this manner follows.

Required the value of .07,142857, C.

	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
.0			
.07	= 7	13	7.04
.001	=	1	12.67
.0004	=		11.46
.00002	=		.57
.000008	=		.22

Value 7 15 15.96 nearly.

EXAMPLE III.

Required the value of .86216517,857142, C.

	<i>Q</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
.8	= 3	5	9	9.6
.06	=	6	11	8.32
.002	=		3	9.34
.0001	=			2.86
.00006	=			1.72
.000005	=			.14

Value 3 12 8 15.98 nearly.

Value 3 12 9 complete.

3. TROY.

3. TROY-WEIGHT.

EXAMPLE I.

Required the value of .778125 lb.

	oz.	dw.	gr.
.7	= 8	8	
.07		16	19.2
.008	=	1	22.08
.0001	=		.576
.00002	=		.1152
.000005	=		.0288

Value 9 6 18.0000 complete.

EXAMPLE II.

Required the value of .43 ounce.

	dw.	gr.
.43	= 8	14.4
.003	=	1.44
.0003	=	.144
.00003	=	.0144

Value 8 16.0 complete.

EXAMPLE III.

Required the value of .5269097 lb.

	oz.	dw.	gr.
.52	= 6	4	19.2
.006	=	1	10.56
.0009	=		5.18

Value 6 6 10.94 nearly.

Value 6 6 11 complete.

There is no occasion to carry the exemplification further; all the varieties that occur in using the tables being already explained. The tables follow.

I. M O.

I. MONEY.

TABLE I. One pound the integer.

	1.		2.		3.		4.
	s.	s.	d.	f.	d.	f.	f.
1	2		2	1.6		.96	.096
2	4		4	3.2		1.92	.192
3	6		7	0.8		2.88	.288
4	8		9	2.4		3.84	.384
5	10	1	0	0.0	1	0.80	.480
6	12	1	2	1.6	1	1.76	.576
7	14	1	4	3.2	1	2.72	.672
8	16	1	7	0.8	1	3.68	.768
9	18	1	9	2.4	2	0.64	.864

TABLE II. One shilling the integer.

	1.		2.		3.	4.
	d.	f.	d.	f.	f.	f.
1	1	0.8		.48	.048	.0048
2	2	1.6		.96	.096	.0096
3	3	2.4		1.44	.144	.0144
4	4	3.2		1.92	.192	.0192
5	6	0.0		2.40	.240	.0240
6	7	0.8		2.88	.288	.0288
7	8	1.6		3.36	.336	.0336
8	9	2.4		3.84	.384	.0384
9	10	3.2	1	0.32	.432	.0432

VOL. II.

I

2. AVOIR.

2. AVOIRDupois-WEIGHT.

TABLE I. One tun the integer.

	1.	2.	3.	4.	5.	6.
	C.	C. $\mathcal{Q}$. lb. oz.	lb. oz.	lb. oz.	oz.	oz.
1	2	22 6.4	2 3.84	3.584	.3584	.03584
2	4	1 16 12.8	4 7.68	7.168	.7168	.07168
3	6	2 11 3.2	6 11.52	10.752	1.0752	.10752
4	8	3 5 9.6	8 15.36	14.336	1.4336	.14336
5	10	1 0 0 0.0	11 3.2	1 1.920	1.7920	.1792
6	12	1 0 22 6.4	13 7.04	1 5.504	2.1504	.21504
7	14	1 1 16 12.8	15 10.88	1 9.088	2.5088	.25088
8	16	1 2 11 3.2	17 14.72	1 12.672	2.8672	.28672
9	18	1 3 5 9.6	20 2.56	2 0.256	3.2256	.32256

TABLE II. One C: or 112 lb. the integer.

	1.	2.	3.	4.	5.	6.
	$\mathcal{Q}$. lb. oz. dr.	lb. oz. dr.	lb. oz. dr.	oz. dr.	dr.	dr.
1	11 3 3.2	1 1 14.72	1 12.672	2.8672	.28672	.028672
2	22 6 6.4	2 3 13.44	3 9.344	5.7344	.57344	.057344
3	1 5 9 9.6	3 5 12.16	5 6.016	8.6016	.86016	.086016
4	1 16 12.8	4 7 10.88	7 2.688	11.4688	1.14688	.114688
5	2 0 0 0.0	5 9 9.6	8 15.360	14.3360	1.43360	.143360
6	2 11 3 3.2	6 11 8.32	10 12.032	1 1.2032	1.72032	.172032
7	2 22 6 6.4	7 13 7.04	12 8.704	1 4.0704	2.00704	.200704
8	3 5 9 9.6	8 15 5.76	14 5.376	1 6.9376	2.29376	.229376
9	3 16 12 12.8	10 1 4.48	1 0 2.048	1 9.8048	2.58048	.258048

TABLE III. One lb. the integer.

	1.	2.	3.	4.
	oz dr.	oz. dr.	dr.	dr.
1	1 9.6	2 .56	.256	.0256
2	3 3.2	5 .12	.512	.0512
3	4 12.8	7 .68	.768	.0768
4	6 6.4	10 .24	1 .024	.1024
5	8 0.0	12 .8	1 .28	.128
6	9 9.6	15 .36	1 .536	.1536
7	11 3.2	1 1 .92	1 .792	.1792
8	12 12.8	1 4.48	2 .048	.2048
9	14 6.4	1 7 .04	2 .304	.2304

3. TROY.

3. TROY-WEIGHT.

TABLE I. One lb. the integer.

	1.	2.	3.	4.
	oz. dw.	oz. dw. gr.	dw. gr.	gr.
1	1 4	2 9.6	5.76	.576
2	2 8	4 19.2	11.52	1.152
3	3 12	7 4.8	17.28	1.728
4	4 16	9 14.4	23.04	2.304
5	6 0	12	1 4.8	2.88
6	7 4	14 9.6	1 10.56	3.456
7	8 8	16 19.2	1 16.32	4.032
8	9 12	19 4.8	1 22.08	4.608
9	10 16	1 1 14.4	2 3.84	5.184

TABLE II. One ounce the integer.

	1.	2.	3.	4.
	dw	dw. gr.	gr.	gr.
1	2	4.8	.48	.048
2	4	9.6	.96	.096
3	6	14.4	1.44	.144
4	8	19.0	1.92	.192
5	10	1 0.0	2.40	.240
6	12	1 4.8	2.88	.288
7	14	1 9.6	3.36	.336
8	16	1 14.4	3.84	.384
9	18	1 19.2	4.32	.432

4. APOTHECARIES WEIGHT.

TABLE. One lb the integer.

	1.				2.				3.	4.
	3	3	3	gr.	3	3	3	gr.	gr.	gr.
1	1	1	1	16			2	17.6	5.76	.576
2	2	3	0	12	1	2	15.2		11.52	1.152
3	3	4	2	8	2	2	12.8		17.28	1.728
4	4	6	1	4	3	2	10.4	1	3.04	2.304
5	6	0	0	0	4	2	8.0	1	8.8	2.880
6	7	1	1	16	5	2	5.6	1	14.56	3.456
7	8	3	0	12	6	2	3.2	2	0.32	4.032
8	9	4	2	8	7	2	0.8	2	6.08	4.608
9	10	6	1	4	1	0	18.4	2	11.84	5.184

5. WOOL WEIGHT.

TABLE I. One last the integer.

	1.			2.		3.	4.
	Sac.	ft.	lb.	S.	ft.	ft. lb.	lb.
1	1	5	2.8	3	1.68	4.368	.4368
2	2	10	5.6	6	3.36	8.736	.8736
3	3	15	8.4	9	5.04	13.104	1.3104
4	4	20	11.2	12	6.72	1 3.472	1.7472
5	6	0	0.0	15	8.40	1 7.840	2.1840
6	7	5	2.8	18	10.08	1 12.208	2.6208
7	8	10	5.6	21	11.76	2 2.576	3.0576
8	9	15	8.4	24	13.44	2 6.944	3.4944
9	10	20	11.2	1 2	1.12	2 11.312	3.9312

TABLE

TABLE II. One sack the integer.

	1. C		2. S		3.	4.
	St.	lb.	St.	lb.	lb.	lb.
1	2	8.4		3.64	.364	.0364
2	5	1.8		7.28	.728	.0728
3	7	1.2	10	.92	1.092	.1092
4	10	5.6	1	0.56	1.456	.1456
5	13	0.0	1	4.20	1.820	.1820
6	15	8.4	1	7.84	2.184	.2184
7	18	2.8	1	11.48	2.548	.2548
8	20	1.2	2	1.12	2.912	.2912
9	23	5.6	2	4.76	3.276	.3276

6. DRY MEASURE

TABLE I. One load the integer.

	1		2.		3.	4.
	Qrs. bush.		Bush. gall.		Galk.	Gall.
1	4		3.2		.32	.032
2	1	0	6.4		.64	.064
3	1	4	1	1.6	.96	.096
4	2	0	1	4.8	1.28	.128
5	2	4	2	0.0	1.60	.160
6	3	0	2	3.2	1.92	.192
7	3	4	2	6.4	2.24	.224
8	4	0	3	1.6	2.56	.256
9	4	4	3	4.8	2.88	.288

TABLE

TABLE II. One quarter the integer.

	1.			2.		3.	4.
	<i>Buf. gal. pts.</i>			<i>Gal. pts.</i>		<i>Pts.</i>	<i>Pts.</i>
1	6	3.2		5.12		.512	.0512
2	1 4	6.4		1 2.24		1.024	.1024
3	2 3	1.6		1 7.36		1.536	.1536
4	3 1	4.8		2 4.48		2.048	.2048
5	4 0	0.0		3 1.60		2.560	.2560
6	4 6	3.2		3 6.72		3.072	.3072
7	5 4	6.4		4 3.84		3.584	.3584
8	6 3	1.6		5 0.96		4.096	.4096
9	7 1	4.8		5 6.08		4.608	.4608

TABLE III. One bushel the integer.

	1.		2.	3.	4.
	<i>Gall pts.</i>		<i>Pts.</i>	<i>Pts.</i>	<i>Pts.</i>
1	6.4		.64	.064	.0064
2	1 4.8		1.28	.128	.0128
3	2 3.2		1.92	.192	.0192
4	3 1.6		2.56	.256	.0256
5	4 0.0		3.20	.320	.0320
6	4 6.4		3.84	.384	.0384
7	5 4.8		4.48	.448	.0448
8	6 3.2		5.12	.512	.0512
9	7 1.6		5.76	.576	.0576

7. WINE.

7. WINE-MEASURE.

TABLE I. One tun the integer.

	1.			2.		3.		4.
	<i>Hhds. gall. pts.</i>			<i>Gall. pts.</i>		<i>Gal. pts.</i>		<i>Pts.</i>
1	25	1.6		2	4.16	2.016		.2016
2	50	3.2		5	0.32	4.032		.4032
3	1 12	4.8		7	4.48	6.048		.6048
4	1 37	6.4		10	0.64	1 0.064		.8064
5	2 0	0.0		12	4.80	1 2.080		1.0080
6	2 25	1.6		15	0.96	1 4.096		1.2096
7	2 50	3.2		17	5.12	1 6.112		1.4112
8	3 12	4.8		20	1.28	2 0.128		1.6128
9	3 37	6.4		22	5.44	2 2.144		1.8144

TABLE II. One hoghead the integer.

	1.		2.		3.		4.
	<i>Gal. pts.</i>		<i>Gal. pts.</i>		<i>Pts.</i>		<i>Pts.</i>
1	6	2.4		5.04	.504		.0504
2	12	4.8	1	2.08	1.008		.1008
3	18	7.2	1	7.12	1.512		.1512
4	25	1.6	2	4.16	2.016		.2016
5	31	4.0	3	1.20	2.520		.2520
6	37	6.4	3	6.24	3.024		.3024
7	43	8.8	4	3.28	3.528		.3528
8	50	3.2	5	0.32	4.032		.4032
9	56	5.6	5	5.36	4.536		.4536

8. BEER.

8. BEER-MEASURE.

TABLE I. One tun the integer.

	1.			2.		3.		4.
	<i>Hbds gal. pts.</i>			<i>Gall. pts.</i>		<i>Gal. pts.</i>		<i>Pts.</i>
1	21	4.8		2	1.28		1.728	.1728
2	43	1.6		4	2.56		3.456	.3456
3	1	10	6.4	6	3.84		5.184	.5184
4	1	32	3.2	8	5.12		6.912	.6912
5	2	0	0.0	10	6.40	1	0.640	.8640
6	2	21	4.8	12	7.68	1	2.368	1.0368
7	2	43	1.6	15	0.96	1	4.096	1.2096
8	3	10	6.4	17	2.24	1	5.824	1.3824
9	3	32	3.2	19	3.52	1	7.552	1.5552

TABLE II. One hoghead the integer.

	1.		2.		3.	4.
	<i>Gal. pts.</i>		<i>Gall. pts.</i>		<i>Pts.</i>	<i>Pts.</i>
1	5	3.2		4.32	.432	.0432
2	10	6.4	1	0.64	.864	.0864
3	16	1.6	1	4.96	1.296	.1296
4	21	4.8	2	1.28	1.728	.1728
5	27	0.0	2	5.60	2.160	.2160
6	32	3.2	3	1.92	2.592	.2592
7	37	6.4	3	6.24	3.024	.3024
8	43	1.6	4	2.56	3.456	.3456
9	48	4.8	4	6.88	3.888	.3888

TABLE

TABLE III. One hoghead, of 51 gallons 8 pints each, the integer.

	1.		2.		3.		4.	
	<i>Gal. pts.</i>		<i>Gal. pts.</i>		<i>Pints.</i>		<i>Pints.</i>	
1	5	0.8		4.08		.408		.0408
2	10	1.6	1	0.16		.816		.0816
3	15	2.4	1	4.24	1	.224		.1224
4	20	3.2	2	0.32	1	.632		.1632
5	25	4.0	2	4.40	2	.040		.2040
6	30	4.8	3	0.48	2	.448		.2448
7	35	5.6	3	4.56	2	.856		.2856
8	40	6.4	4	0.64	3	.264		.3264
9	45	7.2	4	4.72	3	.672		.3672

TABLE IV. One Scots hoghead the integer.

	1.		2.		3.		4.	
	<i>Gal. pts.</i>		<i>Gal. pts.</i>		<i>Pints.</i>		<i>Pints.</i>	
1	1	4.8		1.28		.128		.0128
2	3	1.6		2.56		.256		.0256
3	4	6.4		3.84		.384		.0384
4	6	3.2		5.12		.512		.0512
5	8	0.0		6.40		.640		.0640
6	9	4.8		7.68		.768		.0768
7	11	1.6	1	0.96		.896		.0896
8	12	6.4	1	2.24	1	.024		.1024
9	14	3.2	1	3.52	1	.152		.1152

9. CLOTH-MEASURE.

TABLE I. One yard the integer.

	I.	2.	3.	4
	<i>Qrs n.</i>	<i>N.</i>	<i>N.</i>	<i>N.</i>
1	1.6	.16	.016	.0016
2	3.2	.32	.032	.0032
3	1 0.8	.48	.048	.0048
4	1 2.4	.64	.064	.0064
5	2 0.0	.80	.080	.0080
6	2 1.6	.96	.096	.0096
7	2 3.2	1.12	.112	.0112
8	3 0.8	1.28	.128	.0128
9	3 2.4	1.44	.144	.0144

TABLE II. One ell Flemish the integer.

	1	2	3.	4.
	<i>Qrs n.</i>	<i>N.</i>	<i>N.</i>	<i>N.</i>
1	1.2	.12	.012	.0012
2	2.4	.24	.024	.0024
3	3.6	.36	.036	.0036
4	1 0.8	.48	.048	.0048
5	1 2.0	.60	.060	.0060
6	1 3.2	.72	.072	.0072
7	2 0.4	.84	.084	.0084
8	2 1.6	.96	.096	.0096
9	2 2.8	.08	.108	.0108

TABLE

TABLE III. One ell English the integer.

	1.		2.	3	4.
	<i>Qrs. n.</i>		<i>N.</i>	<i>N.</i>	<i>N.</i>
1		2	.2	.02	.002
2	1	0	.4	.04	.004
3	1	2	.6	.06	.006
4	2	0	.8	.08	.008
5	2	2	1.0	.10	.010
6	3	0	1.2	.12	.012
7	3	2	1.4	.14	.014
8	4	0	1.6	.16	.016
9	4	2	1.8	.18	.018

10. LONG MEASURE.

TABLE I. One mile the integer.

	1.		2.		3	4.
	<i>Fur.</i>	<i>ch.</i>	<i>Ch.</i>	<i>p. l</i>	<i>P. l</i>	<i>L.</i>
1		8		3 5	8	.8
2	1	6	1	2 10	16	1.6
3	2	4	2	1 15	24	2.4
4	3	2	3	0 20	1 7	3.2
5	4	0	4	0 0	1 15	4.0
6	4	8	4	3 5	1 23	4.8
7	5	6	5	2 10	2 6	5.6
8	6	4	6	1 15	2 14	6.4
9	7	2	7	0 20	2 22	7.2

TABLE II. One chain the integer.

	1.	2.	3.	4.
	<i>P. I.</i>	<i>L.</i>	<i>L.</i>	<i>L.</i>
1	10	1	.1	.01
2	20	2	.2	.02
3	30	3	.3	.03
4	40	4	.4	.04
5	50	5	.5	.05
6	60	6	.6	.06
7	70	7	.7	.07
8	80	8	.8	.08
9	90	9	.9	.09

TABLE III. One pole the integer.

	1.	2.	3.	4.
	<i>Yds ft. inc.</i>	<i>Ft. inch.</i>	<i>Inch.</i>	<i>Inch.</i>
1	1 7.8	1.98	.198	.0198
2	1 0 3.6	3.96	.396	.0396
3	1 1 11.4	5.94	.594	.0594
4	2 0 7.2	7.92	.792	.0792
5	2 2 3.0	9.90	.990	.0990
6	3 0 10.8	11.88	1.188	.1188
7	3 2 6.6	13.86	1.386	.1386
8	4 1 2.4	15.84	1.584	.1584
9	4 2 10.2	17.82	1.782	.1782

TABLE

TABLE IV. One yard the integer.

	1.	2.	3.	4.
	<i>Ft. in.</i>	<i>Inch.</i>	<i>Inch.</i>	<i>Inch.</i>
1	3.6	.36	.036	.0036
2	7.2	.72	.072	.0072
3	10.8	1.08	.108	.0108
4	14.4	1.44	.144	.0144
5	18.0	1.80	.180	.0180
6	21.6	2.16	.216	.0216
7	25.2	2.52	.252	.0252
8	28.8	2.88	.288	.0288
9	32.4	3.24	.324	.0324

TABLE V. One foot the integer.

	1.	2.	3.	4.
	<i>In. lin.</i>	<i>In. lin.</i>	<i>Lin.</i>	<i>Lin.</i>
1	2.4	1.44	.144	.0144
2	4.8	2.88	.288	.0288
3	7.2	4.32	.432	.0432
4	9.6	5.76	.576	.0576
5	12.0	7.20	.720	.0720
6	14.4	8.64	.864	.0864
7	16.8	10.08	1.008	.1008
8	19.2	11.52	1.152	.1152
9	21.6	12.96	1.296	.1296

TABLE

TABLE VI. One Scots mile the integer.

1.	2.	3.	4.
<i>Fur. ch.</i>	<i>Ch fa. ell</i>	<i>Fa. el.</i>	<i>Ells.</i>
1 8	3 1.2	1.92	.192
2 6	1 2 2.4	3.84	.384
3 4	2 1 3.6	5.76	.576
4 2	3 0 4.8	1 1.68	.768
5 0	4 0 0.0	1 3.60	.960
6 8	4 3 1.2	1 5.52	1.152
7 6	5 2 2.4	2 1.44	1.344
8 4	6 1 3.6	2 3.36	1.536
9 2	7 0 4.8	2 5.28	1.728

II. LAND-MEASURE.

TABLE I. One acre the integer.

1.	2.	3.	4.
<i>R. sq. p.</i>	<i>Sq p. sq. yd.</i>	<i>Sq. p. sq. y.</i>	<i>Sq. yd.</i>
1 16	1 18.15	4.84	.484
2 32	3 6.05	9.68	.968
3 8	4 24.20	14.52	1.452
4 24	6 12.10	19.36	1.936
5 0	7 30.25	24.20	2.420
6 16	9 18.15	29.04	2.904
7 32	11 6.05	1 3.63	13.388
8 8	12 24.20	1 8.47	3.872
9 24	14 12.10	1 13.31	4.356

TABLE

TABLE II. One Scots acre the integer.

	1.	2	3.	4.
	<i>R. sq f.</i>	<i>Sq. f /q. el.</i>	<i>Sq. f. sq. el.</i>	<i>Sq. el.</i>
1	16	1 21.6	5.76	.576
2	32	3 6.2	11.52	1.152
3	1 8	4 27.8	17.28	1.728
4	1 24	6 13.4	23.04	2.304
5	2 0	7 35.0	28.80	2.880
6	2 16	9 20.6	34.56	3.456
7	2 32	11 42.2	1 4.32	4.032
8	3 8	12 28.8	1 10.08	4.608
9	3 24	14 13.4	1 15.84	5.184

12. SUPERFICIAL MEASURE.

TABLE I. One square yard the integer.

	1.	2.	3.	4.
	<i>S. ft. sq. in.</i>	<i>Sq. in.</i>	<i>Sq. in.</i>	<i>Sq. in.</i>
1	129.6	12.96	1.296	.1296
2	1 115.2	25.92	2.592	.2592
3	2 100.8	38.88	3.888	.3888
4	3 86.4	51.84	5.184	.5184
5	4 72.0	64.80	6.480	.6480
6	5 57.6	77.76	7.776	.7776
7	6 43.2	90.72	9.072	.9072
8	7 28.8	103.68	10.368	1.0368
9	8 14.4	116.64	11.664	1.1664

TABLE

TABLE II. One square foot the integer.

	I.		2.		3.		4.
	<i>Sq. in. sq. li.</i>		<i>S. in. sq. li.</i>		<i>S. in. sq. lin.</i>		<i>Sq. lin.</i>
1	14 57.6	1	63.36		20.736		2.0736
2	28 115.2	2	126.72		41.472		4.1472
3	43 28.8	4	46.08		62.208		6.2208
4	57 86.4	5	109.44		82.944		8.2944
5	72 00.0	7	28.80		103.680		10.3680
6	86 57.6	8	92.16		124.416		12.4416
7	100 115.2	10	11.52	1	01.152		14.5152
8	115 28.8	11	74.88	1	21.888		16.5888
9	129 86.4	12	138.24	1	42.624		18.6624

13. SOLID MEASURE.

TABLE I. One cubic yard the integer.

	I.		2.		3.		4.
	<i>C. ft. c. in.</i>		<i>C. f. c. in.</i>		<i>C. in.</i>		<i>C. in.</i>
1	2 1209.6		466.56		46.656		4.6656
2	5 691.2		933.12		93.312		9.3312
3	8 172.8		1399.68		139.968		13.9968
4	10 1382.4	1	138.24		186.624		18.6624
5	13 864.0	1	604.80		233.280		23.3280
6	16 345.6	1	1071.36		279.936		27.9936
7	18 1555.2	1	1537.92		326.592		32.6592
8	21 1036.8	2	276.48		373.248		37.3248
9	24 518.4	2	743.04		419.904		41.9904

TABLE II. One cubic foot the integer.

	1.	2.	3.	4.
	<i>C. in. c. lin.</i>	<i>C. in. c. lin.</i>	<i>C. in. c. lin.</i>	<i>C. in. c. lin.</i>
1	172 1382.4	17 483.84	1 1257.984	298.5984
2	345 1036.8	34 967.68	3 787.968	597.1968
3	518 691.2	51 1451.52	5 317.952	895.7952
4	691 345.6	69 207.36	6 1575.936	1194.3936
5	864 000.0	86 691.20	8 1105.920	1492.9920
6	1036 1382.4	103 1175.04	10 635.904	1 63.5904
7	1209 1036.8	120 1628.88	12 163.888	1 362.1888
8	1382 691.2	138 414.72	13 1423.872	1 660.7872
9	1555 345.6	155 898.56	15 953.856	1 959.3856

14. A CIRCLE.

TABLE I. One circle the integer.

	1.	2.	3.	4.	5.	6.
	<i>Si. de.</i>	<i>Si. de. m.</i>	<i>Deg. mi.</i>	<i>Min.</i>	<i>Min.</i>	<i>Min.</i>
1	1 6	3 36	21 .6	2.16	.216	.0216
2	2 12	7 12	43 .2	4.32	.432	.0432
3	3 18	10 48	1 4 .8	6.48	.648	.0648
4	4 24	14 24	1 26 .4	8.64	.864	.0864
5	6 0	18 00	1 48 .0	10.80	1.080	.1080
6	7 6	21 36	2 9 .6	12.96	1.296	.1296
7	8 12	25 12	2 31 .2	15.12	1.512	.1512
8	9 18	28 48	2 52 .8	17.28	1.728	.1728
9	10 24	1 2 24	3 14 .4	19.44	1.944	.1944

TABLE II. One sign the integer.

	1.	2.	3.	4.	5.	6.
	Deg.	Deg.min.	Min. sec.	Min. sec.	Sec.	Sec.
1	3	18	1 48	10.8	1.08	.108
2	6	36	3 36	21.6	2.16	.216
3	9	54	5 24	32.4	3.24	.324
4	12	1 12	7 12	43.2	4.32	.432
5	15	1 30	9 0	54.0	5.40	.540
6	18	1 48	10 48	1 4.8	6.48	.648
7	21	2 6	12 36	1 15.6	7.56	.756
8	24	2 24	14 24	1 26.4	8.64	.864
9	27	2 42	16 12	1 37.2	9.72	.972

15. T I M E.

TABLE I. One year of 365 days the integer.

	1.	2.	3.	4.
	Days.	Days.	Days.	Days.
1	36.5	3.65	.365	.0365
2	73.0	7.30	.730	.0730
3	109.5	10.95	1.095	.1095
4	146.0	14.60	1.460	.1460
5	182.5	18.25	1.825	.1825
6	219.0	21.90	2.190	.2190
7	255.5	25.55	2.555	.2555
8	292.0	29.20	2.920	.2920
9	328.5	32.85	3.285	.3285

TABLE

TABLE II. One year of 13 months, 28 days each, the integer.

	I.	2.	3.	4.
	<i>M.w.d. h.</i>	<i>M.w.d. h.</i>	<i>D. h.</i>	<i>Hours.</i>
1	1 1 1 9.6	3 15.36	8.736	.8736
2	2 2 2 19.2	1 0 6.72	17.472	1.7472
3	3 3 4 4.8	1 3 22.08	1 2.208	2.6208
4	5 0 5 14.4	2 0 13.44	1 10.944	3.4944
5	6 2 0 0.0	2 4 4.80	1 19.680	4.3680
6	7 3 1 9.6	3 0 20.16	2 4.416	5.2416
7	9 0 2 19.2	3 4 11.52	2 13.152	6.1152
8	10 1 4 4.8	1 0 1 2.88	2 21.888	6.9888
9	11 2 5 14.4	1 0 4 18.24	3 6.624	7.8624

TABLE III. One month of 28 days the integer.

	I.	2.	3.	4.
	<i>W.d. h.</i>	<i>D. h.</i>	<i>H.</i>	<i>H.</i>
1	2 19.2	6.72	.672	.0672
2	5 14.4	13.44	1.344	.1344
3	1 1 9.6	20.16	2.016	.2016
4	1 4 4.8	1 2.88	2.688	.2688
5	2 0 0.0	1 9.60	3.360	.3360
6	2 2 19.2	1 16.32	4.032	.4032
7	2 5 14.4	1 23.04	4.704	.4704
8	3 1 9.6	2 5.76	5.376	.5376
9	3 4 4.8	2 12.48	6.048	.6048

TABLE IV. One day the integer.

	1.	2.	3.	4.
	<i>H. m.</i>	<i>H. m.</i>	<i>Min.</i>	<i>Min.</i>
1	2 24	14.4	1.44	.144
2	4 48	28.8	2.88	.288
3	7 12	43.2	4.32	.432
4	9 36	57.6	5.76	.576
5	12 00	1 12.0	7.20	.720
6	14 24	1 26.4	8.64	.864
7	16 48	1 40.8	10.08	1.008
8	19 12	1 55.2	11.52	1.152
9	21 36	2 9.6	12.96	1.296

TABLE V. One hour, or one degree, the integer.

	1.	2.	3.	4.
	<i>Min.</i>	<i>M. sec.</i>	<i>Sec.</i>	<i>Sec.</i>
1	6	36	3.6	.36
2	12	1 12	7.2	.72
3	18	1 48	10.8	1.08
4	24	2 24	14.4	1.44
5	30	3 00	18.0	1.80
6	36	3 36	21.6	2.16
7	42	4 12	25.2	2.52
8	48	4 48	28.8	2.88
9	54	5 24	32.4	3.24

PROB.

P R O B. V.

To reduce a decimal to its primitive vulgar fraction.

C A S E I.

When the given decimal is finite.

R U L E.

Divide both numerator and denominator of the given decimal by their greatest common measure; the quot is the vulgar fraction required.

Thus, $.875 = \frac{875}{1000} = \frac{7}{8}$. For $875)1000(1$

Greatest common measure $125 \overline{)875(7}$
 $\frac{875}{875}$
 (0)

And $125 \overline{)1000(8}$.

Or, instead of dividing by the greatest common measure, you may divide by 2, 5, 10, or by any number that will divide both numerator and denominator without a remainder, continuing the operation till the fraction be reduced to its lowest terms.

Thus, $.0625 = \frac{625}{10000} = \frac{1}{16}$. For $5 \overline{)625(125}$ $5 \overline{)125(25}$ $5 \overline{)25(5}$ $5 \overline{)5(1}$
 $\frac{625}{10000} \mid \frac{125}{2000} \mid \frac{25}{400} \mid \frac{5}{80} \mid \frac{1}{16}$

C A S E II.

When the given decimal is a pure repeater, or a pure circulate.

R U L E.

Make the repeating figure, or the figures of the circle, the numerator of the vulgar fraction; the denominator is 9 for the repeating figure, or 9 for every figure of the circle; and then, if occasion require, reduce this fraction to its lowest terms.

Thus, $.1 = \frac{1}{9} = \frac{1}{9}$, and $.6 = \frac{6}{9} = \frac{2}{3}$, and $.5 = \frac{5}{9}$.

Again, $.27 = \frac{27}{99} = \frac{3}{11}$, and $.714285 = \frac{714285}{999999} = \frac{5}{7}$.

C A S E

C A S E III.

When the given decimal is a mixt repeater, or a mixt circulate.

R U L E.

From the mixt repeater, or mixt circulate, subtract the finite part, and the remainder is the numerator of the vulgar fraction; the denominator is 9 for the repeating figure, or 9 for every figure of the circle, with as many eiphers annexed as there are figures in the finite part.

Thus, $.0\bar{3} = \frac{3}{90} = \frac{1}{30}$, and $.1\bar{6} = \frac{16}{90} = \frac{8}{45}$, and $.08\bar{3} = \frac{75}{900} = \frac{5}{60} = \frac{1}{12}$.

Again, $.0,45 = \frac{45}{990} = \frac{1}{22}$, and $.3,18 = \frac{315}{990} = \frac{7}{22}$, and $.03,571428 = \frac{3571425}{9999990} = \frac{1}{28}$.

The reason of the rule may be shewn thus: Esteem the finite part of the last example an integer, and then the mixt number $3\frac{571428}{999999}$ will be equal to the given circulate. Again; reduce this mixt number to an improper fraction, viz. multiply the integer 3 by the denominator 999999, and to the product add the numerator, as directed in reduction of vulgar fractions.

Multiply the integer 3 into 999999 by the method of multiplying any number by 9, 99, 999, &c. taught in multiplication of integers, and to the product add the numerator, and the sum shall be the numerator of the improper fraction, as in the margin.

3000000

3

2999997

571428

3571425 num.

Now it is evident that the same numerator will be found, if, in the upper line, instead of the six ciphers, I place the figures of the circle, and from them subtract 3, the finite part.

3571428

3

3571425 num.

To the numerator thus found, the denominator is 999999; and so the vulgar fraction is $\frac{3571428}{999999}$. But we esteemed 3 an integer; whereas, in fact, it is $\frac{3}{100}$; and

and so our vulgar fraction will be 100 times greater than it ought to be : to correct this error we must multiply the denominator by 100, which is done by annexing two ciphers to it ; and the true fraction comes out to be $\frac{3571428}{99999900}$, as by the rule.

Because this rule is of great importance, and will often occur in practice, I shall here subjoin some more examples.

Ex. 1. Reduce .0416 to a vulgar fraction.

$$\begin{array}{r} .0416 \\ \underline{41} \\ \text{Num. } 375 \\ \text{Den. } 9000 \end{array} = \frac{1}{24}$$

Ex. 2. Reduce .583 to a vulgar fraction.

$$\begin{array}{r} .583 \\ \underline{58} \\ \text{Num. } 525 \\ \text{Den. } 900 \end{array} = \frac{7}{12}$$

Ex. 3. Reduce .1,153846, to a vulgar fraction.

$$\begin{array}{r} .1,153846, \\ \underline{1} \\ \text{Num. } 153845 \\ \text{Den. } 9999990 \end{array} = \frac{3}{26}$$

Ex. 4. Reduce .53,571428, to a vulgar fraction.

$$\begin{array}{r} .53,571428, \\ \underline{53} \\ \text{Num. } 53571375 \\ \text{Den. } 99999900 \end{array} = \frac{15}{28}$$

In this manner too may any mixt number, consisting of an integer with a repeater or circulate, be reduced to an improper vulgar fraction ; but no ciphers are to be annexed to the denominator for the figures of the integer.

Ex.

Ex. 1. Reduce $8.\overline{3}$ to an improper vulgar fraction.

$$\begin{array}{r} 8.\overline{3} \\ \underline{8} \\ \text{Num. } 75 \\ \text{Den. } 9 \end{array} = \frac{25}{3} = 8\frac{1}{3}$$

Ex. 2. Reduce $4.1\overline{6}$ to an improper vulgar fraction.

$$\begin{array}{r} 4.1\overline{6} \\ \underline{41} \\ \text{Num. } 375 \\ \text{Den. } 90 \end{array} = \frac{75}{18} = \frac{25}{6} = 4\frac{1}{6}$$

Ex. 3. Reduce $65.296,$ to an improper vulgar fraction.

$$\begin{array}{r} 65.296, \\ \underline{65} \\ \text{Num. } 65231 \\ \text{Den. } 999 \end{array} = 65\frac{8}{27}$$

Ex. 4. Reduce $8.32,142857,$ to an improper vulgar fraction.

$$\begin{array}{r} 8.32,142857, \\ \underline{832} \\ \text{Num. } 832142025 \\ \text{Den. } 999999000 \end{array} = 8\frac{9}{28}$$

Approximate decimals being imperfect, cannot be exactly reduced back to the vulgar fractions from which they resulted. But if the approximate be completed by annexing to it a vulgar fraction, whereof the remainder of the division is the numerator, and the divisor the denominator, you shall have a mixt number; which you may reduce to an improper vulgar fraction; then to the denominator annex as many ciphers as there are figures in the approximate; and this fraction, reduced

reduced to its lowest terms, will be the primitive vulgar fraction required.

P R O B. VI.

To reduce unlike circles to others that are similar and conterminous.

Similar or like circles are such as consist of an equal number of places.

Thus, .27, and .09, are similar circles, as consisting of two places each. But .63, and .148, are unlike; the former consisting of two, and the latter of three places.

Conterminous circles are such as begin and end at the same distance from the decimal point.

Thus, .153846, and .384615, are conterminous; because they both begin at the place of primes, and have an equal number of places. And .0,714285, and .7,857142, are conterminous, because they both begin at the place of seconds, and have the same number of places. But .81, and .1,36, are not conterminous, the former beginning at the place of primes, and the latter at the place of seconds. Again, .63, and .481, are not conterminous, because they have not the same number of places; for circles cannot be conterminous unless they be at the same time similar.

Unlike circles are reduced to similar ones by the following

R U L E.

Find the least multiple of the numbers denoting the number of places in the several given circles, and extend each of the given circles to as many places as there are units in the least multiple.

Thus, to reduce the unlike circles

.63, and .148, to similar ones, I extend both circles to six places, because 6 is the least multiple of 2 and 3, the number of places in the given circles.

Again, to reduce .72, and .02439, to similar circles, I .72, = .7272727272, extend each of them to ten .02439, = .0243902439, places, because 10 is the least multiple of 2 and 5, the number of places in the given circle.

In a circle any one of the circulating figures may be made the first of the circle. Thus, 7.592, may be expressed thus, 7.5,925,; or thus, 7.59,259,; and that without changing its value: consequently a pure circulate may put on the form of a mixt circulate, if one or more figures on the left be set aside for the finite part; thus, .72, = .7,27, where .7, is the finite part; and thus, .57,1428, = .57,142857, and thus, .36, = .363,63,

That the value is not changed may be thus demonstrated.

$$.7,27, = \frac{720}{990} = \frac{72}{99} = .72,$$

Hence two or more given circles may be made conterminous, by the following

R U L E.

Set aside by a comma on the left, as many figures as there are places in the longest finite part, and then prolong the several circles to as many places as will make them similar.

Ex. 1. To make .54,63, .54,63, = .54,636363, and .9,148, conterminous. .9,148, = .91,481481,

Here, because .54, the longest finite part, consists of two places, I set aside .91, in the other circulate, for a finite part, and then I prolong both circles to six places, which renders them similar.

Ex. 2. To make the three circles in the margin .17,857142, = .17,857142, conterminous. .592, = .59,259259,

Here, because the first .54, = .54,545454, circulate has a finite part of two places, I set off two figures on the left from each of the other two circles, and then I prolong each circle to six places, which renders them similar.

Ex.

Ex. 3. To make the three following circles conterminous.

Here, because the first circulate has a finite part of three places, I set off three figures in each of the other two for finite parts, and then, to render them similar, I extend the figures of the circles to six places.

$$\begin{array}{l} .714,285714, = .714,285714, \\ .3,571428, = .357,142857, \\ .36, = .363,636363, \end{array}$$

C H A P. III.

ADDITION OF DECIMALS.

RULE I. Place the given decimals so that the points may stand directly under each other, and consequently tenths under tenths, hundredths under hundredths, &c.; then, if the given decimals be all finite or approximate, add them as integers, inserting the decimal point directly under the column of points. The figures on the left of the point are integers, and those on the right are a decimal of the integer, consisting of as many places as there are figures in the longest of the given decimals.

The operation is the same here as in addition of vulgar fractions; for a cipher on the right of a decimal does not change its value: if, therefore, ciphers be annexed, so as to give every decimal the same number of places, as is done in the margin, they will by this means be reduced to a common denominator, viz. 1000.

Ex. 1.

$$\begin{array}{r} .75 = .750 \\ .895 = .895 \\ .5 = .500 \\ .625 = .625 \\ .725 = .725 \\ \hline 3.495 = 3\frac{495}{1000} \end{array}$$

Ex. 2.

$$\begin{array}{r}
 45\frac{1}{4} = 45.25 \\
 68\frac{1}{8} = 68.125 \\
 72\frac{1}{2} = 72.5 \\
 24\frac{3}{4} = 24.75 \\
 36\frac{1}{10} = 36.05 \\
 28\frac{1}{10} = 28.0625 \\
 \hline
 274\frac{5}{8} = 274.7375
 \end{array}$$

Ex. 3.

L.	s.	d.	L.
42	17	6	= 42.875
8	8	$7\frac{1}{2}$	= 8.43125
12	5	6	= 12.275
28	14	6	= 28.725
7	10	6	= 7.5251
99	16	$7\frac{1}{2}$	= 99.83125

Note. If the decimals to be added are of different denominations, first reduce them to one denomination, and then add. The reason is, because like things only can be added or subtracted.

Ex. What is the sum of .725 l. and .625 s.

Here I may either reduce the decimal of a shilling to that of a pound, or I may reduce the decimal of a pound to that of a shilling.

First, reduce the decimal of a shilling to that of a pound, by reduction-ascending, *viz.* divide by 20, as follows.

$$\begin{array}{r}
 \text{L.} \\
 20).62500(.03125 \\
 \underline{.725}
 \end{array}$$

$$\begin{array}{r}
 \text{s.} \quad \text{d.} \\
 \text{Sum } .75625 = 15 \quad 1\frac{1}{2}
 \end{array}$$

Secondly, reduce the decimal of a pound to that of a shilling, by reduction-descending; that is, multiply by 20, as follows.

The

The answer here
is the same as be-
fore.

$$\begin{array}{r}
 .7251. \\
 \underline{20} \\
 s. 14.500 \\
 \underline{07.625} \\
 s. 15.125 \text{ sum.} \\
 \underline{12} \\
 d. 1.500 \\
 \underline{4} \\
 f. 2.0
 \end{array}$$

APPROXIMATES.

If the decimals to be added run on to a great many places, it will be sufficient in most cases to use only four or five places, and observe to increase the figure at which you break off by an unit, if the rejected figure on the right exceed 5. And in adding such approximates, omit the right-hand figure of the sum, as uncertain, but take in the carriage. Follows an example at large, and the same contracted.

Ex. 4. at large.

contracted.

$$\begin{array}{r}
 12.2352946 \\
 8.15789325 \\
 7.086968435 \\
 6.32143482 \\
 4.75 \\
 \hline
 38.551591105
 \end{array}$$

$$\begin{array}{r}
 12.23529+ \\
 8.15789+ \\
 7.08697- \\
 6.32143+ \\
 4.75 \\
 \hline
 38.5515 \text{ certain.}
 \end{array}$$

RULE II. When all or any of the given decimals are repeaters, give every repeater the same number of places, and one place more than the longest finite; and for every nine in the right-hand column carry 1, or to its sum add 1 for every nine, and then carry at ten.

Ex.

Ex. 1.

$$\begin{array}{r}
 748\frac{1}{3} = 748.33 \\
 653\frac{2}{3} = 653.66 \\
 84\frac{1}{9} = 84.11 \\
 25\frac{5}{8} = 25.83 \\
 37\frac{5}{8} = 37.55 \\
 8\frac{1}{8} = 8.1\bar{6} \\
 \hline
 1557\frac{2}{3} = 1557.66
 \end{array}$$

Ex. 2.

L.	s.	d.	=	L.
26	14	$1\frac{1}{4}$	=	26.7052083
45	10	$7\frac{3}{4}$	=	45.5322916
24	10	$6\frac{3}{4}$	=	24.528125
32	15	0	=	32.75
37	13	4	=	37.6666666
42	18	8	=	42.9333333
210	2	$3\frac{3}{4}$	=	210.1156250

Ex. 3.

s.	d.	=	L.
13	4	=	.66
6	8	=	.33
3	4	=	.16
10	0	=	.5
4	0	=	.2
6	0	=	.3
2	3	4	= 2.16

Ex. 4.

oz.	Troy.
4.35	2083
7.45	625
8.90	833
9.50	333
5.03	75
9.04	7916
44.30	5416

Ex. 5.

Feet.
6.083
5.333
8.416
4.25
7.75
3.916
35.750

Ex. 6.

Days.
342.25416
238.33333
316.41666
45.20833
35.29166
40.125
1017.62916

In Example 1. the sum of the right-hand column is 24, which contains 9 twice, and 6 over; so I set down 6 and carry 2: or to the sum 24 I add 2, for the two nines, which makes 26; so I set down 6 and carry 2. I proceed with the rest as in integers.

The sums, differences, and products of interminate decimals, are always interminate, unless they end in a cipher, as in Ex. 2.

From Ex. 3. it appears, that addition may sometimes, by the means of repeating decimals, be performed with more ease and fewer figures, than by finite numbers.

A repeating digit is the numerator of a vulgar fraction, whose denominator is 9; and hence, in adding a column of repeating digits, every 9 of the sum is 9, or an unit, to be carried; and what is over a just number of nines is so many ninth parts.

Or, if to the sum of a column of repeating digits, 1 for

for every 9 contained in it be added, we then carry 1 for every ten; but what is over a just number of tens will still continue to be ninth parts.

If in any example the repeating figures happen all to be reiterated, the carriage from the right-hand column adjusts the column on the left; or makes every ten of them equal to an unit of the next superior column, &c. Thus, if we imagine a column of the repeating figures reiterated on the right of any example, the carriage from it would adjust the right-hand column of the example.

RULE III. When all or any of the given decimals are circulates, make all the circles conterminous, find the number of tens to be carried from the left-hand column of the circles, add this carriage to the right-hand column, and proceed as in addition of integers.

If repeaters be mixed with the circulates, give the repeaters the form of circulates, by extending the repeating figures till they become conterminous with the other circles; as in Ex. 3. and 4.

If finite decimals are joined with the circulates, extend the finite parts of all the circulates to as many places as there are figures in the longest finite; as in Ex. 4.

E X A M P L E I.

$$\begin{array}{rcl} \frac{3}{7} & = & .428571, = .428571, \\ \frac{6}{7} & = & .857142, = .857142, \\ \frac{5}{11} & = & .45, = .454545, \\ \frac{10}{27} & = & .370, = .370370, \end{array}$$

2.110630

In order to find the carriage from the left-hand column of the circles, I add the column next to it on the right, saying, $7 + 5 + 5 + 2 = 19$; from which I carry 1, and say, $1 + 3 + 4 + 8 + 4 = 20$; from which I carry 2, and go on to add the right-hand column of the circle, saying, the carriage $2 + 5 + 2 + 1 = 10$; so I set down 0, and carry 1, and proceed with the rest as in integers.

The

The adding the carriage from the left-hand column of the circles to the column on the right hand, arises from the flux of numbers; for as the circles repeat infinitely, if we suppose a new set of the same circles to be repeated upon the right of our examples, it is plain, that in adding them the carriage from the left-hand column of the new set would naturally fall into the right-hand column of our example.

The operation here is the same as in addition of vulgar fractions; for every circle is the numerator of a vulgar fraction, whose common denominator is 999999; and if the circles or numerators be added, without minding any carriage from the left-hand column, the sum will be 2110628.

$$\begin{array}{r} \text{And } 999999)2110628(2 \\ \underline{1999998} \end{array}$$

$$110630$$

But, by pointing off from the sum of the circles six figures towards the right, we divide by 1000000, instead of dividing by 999999; which gives indeed the same quot, but makes the remainder too small.

Now, that the carriage-figure from the left-hand column of the circles, is the integral part of the quot, and at the same time the difference betwixt the true and false remainder, is evident; for the quotient-figure 2, multiplied into the two divisors 1000000 and 999999, gives two products, whose difference is 2; and consequently, if the greatest product, viz $2 \times 1000000 = 2000000$, be subtracted from the dividend, the result will want 2 of the true remainder. To prevent such errors, and to put the work on a sure footing, find the carriage from the left-hand column of the circles, add this carriage to the right-hand column, divide the sum by 1000000, and you will have a true quot, and a true remainder. The learner may look back to division of integers, where the method of dividing by 9,99,999, &c. is explained.

Hence

Hence it follows, that if we add the circles as they stand, without minding any carriage from the left, and to the sum add the excrescent figure on the left of the decimal point, we shall have the full sum of the circles, both as to the integral and fractional part, as in the margin.

$$\begin{array}{r}
 .428571, \\
 .857142, \\
 .454545, \\
 \underline{.370370,} \\
 2.110628 \\
 \quad 2 \\
 \hline
 2.110630,
 \end{array}$$

Pure repeaters, being the numerators of vulgar fractions, whose denominator is 9 as often taken as the digit is repeated, may be added in the same manner as circles. But in examples clear of circulates, the method prescribed in Case II. is preferable.

In adding circles and pure repeaters by the method now explained, it will sometimes happen that the fractional part of the sum will be a series of nines, as in the margin: and in this case, the numerator of the fraction being the same with the denominator, its value will be unity; and accordingly 1 must be added to the integral part. But in adding pure repeaters by the method in Case II. this cannot happen.

$$\begin{array}{r}
 .857142, \quad .\textcircled{8}666, \\
 .571428, \quad .\textcircled{5}666, \\
 .857142, \quad .\textcircled{8}666, \\
 \underline{.714285,} \quad \underline{6666,} \\
 2.999999, \quad 1.9999, \\
 \quad 1 \quad \quad \quad \underline{1} \\
 \quad \quad \quad \underline{2.0000} \\
 3.000000
 \end{array}$$

By way of proof, I shall here add all the vulgar fractions in Example I. and reduce their sum to a mixt number, continuing the division to a decimal.

$$\frac{3}{7} + \frac{6}{7} + \frac{5}{11} + \frac{10}{27} = \frac{6237}{14553} + \frac{12474}{14553} + \frac{6615}{14553} + \frac{5390}{14553} = \frac{30716}{14553}$$

$$14553)30716(2.110630,$$

$$29106$$

$$*16100$$

$$14553$$

$$15470$$

$$14553$$

$$91700$$

$$87318$$

$$43820$$

$$43659$$

$$*16100$$

Here the dividial being the same with the second, a new circle begins.

EXAMPLE II.

$$\frac{5}{14} = .3,571428, = .35,714285,$$

$$\frac{1}{28} = .0,384615, = .03,846153,$$

$$\frac{12}{28} = .67,857142, = 67,857142,$$

$$\frac{3}{11} = 27, = .27,272727,$$

$$\frac{4}{37} = .148, = .14,814814,$$

$$1.49,505124,$$

Here I make all the circles conterminous; and having found the carriage from the left-hand column of the circles, I add it up with the right-hand column, and then proceed as in adding integers.

The reason of making the circles conterminous is, because, by this means, like things come to be added; for the finite parts are all tenth parts, but the circles are all ninth parts.

EXAMPLE III.

$$\frac{42}{52} = .94,230769, = .94,230769,$$

$$\frac{278}{390} = 7,128205, = .71,282051,$$

$$\frac{1}{3} = .3 = 33,333333,$$

$$\frac{2}{3} = .6 = .66,666666,$$

$$\frac{1}{6} = .16 = 16,666666,$$

$$\frac{1}{12} = .083 = .08,333333,$$

$$2.90,512820,$$

Here

Here I give the repeaters the form of circulates, and, by extending the repeating figure, I make their circles conterminous with the other circles.

That repeaters reduced to the form of circulates retain their former value, may be thus demonstrated.

$$\begin{array}{r} .33,333333, \\ \quad \quad \quad 33 \\ \hline \text{Num. } 33333300 \\ \text{Den. } 99999900 \end{array} = \frac{333333}{999999} = \frac{3}{9} = \frac{1}{3}$$

EXAMPLE IV.

L.		L.
$18\frac{1}{4} = 18.7,857142,$		$= 18.785,714285,$
$25\frac{9}{11} = 25.81,$		$= 25.818,181818,$
$14\frac{1}{2} = 14.5$		$= 14.5$
$29\frac{3}{4} = 29.75$		$= 29.75$
$35\frac{7}{8} = 35.875$		$= 35.875$
$24\frac{1}{5} = 24.01\bar{2}$		$= 24.01\bar{2},333333,$
$32\frac{7}{9} = 32.\bar{7}$		$= 32.\bar{7},777777,$
		181,520,007215,

Here I extend the finite parts of all the circulates and repeaters to three places, because 875, the longest finite, consists of three places; and by this means the circles, which are all ninth parts, come to be added by themselves.

To ascertain the carriage from the left-hand column of the circles, you must go back to the second column on the right of it, saying $7 + 3 + 1 + 4 = 15$, one carried, &c.

CHAP. IV.

SUBTRACTION OF DECIMALS.

RULE I. Place the minor under the major, so that the points may be in one column; and then, if the given

N 2

decimals

decimals be finite or approximate, work as in subtraction of integers.

If the major and minor have not the same number of places, imagine the void places to be filled up with ciphers.

EXAMPLE I.

$$\begin{array}{r}
 \text{L.} \quad \text{s.} \quad \text{d.} \quad \text{L.} \\
 \text{From } 48 \quad 10 \quad 6 = 48.525 \\
 \text{Sub. } 18 \quad 12 \quad 8\frac{1}{4} = 18.634375 \\
 \hline
 \text{Rem. } 29 \quad 17 \quad 9\frac{3}{4} = 29.890625
 \end{array}$$

EXAMPLE II.

$$\begin{array}{r}
 \text{C.} \quad \text{Q. lb.} \quad \text{C.} \\
 \text{From } 54 \quad 2 \quad 21 = 54.6875 \\
 \text{Sub. } 36 \quad 3 \quad 14 = 36.875 \\
 \hline
 \text{Rem. } 17 \quad 3 \quad 7 = 17.8125
 \end{array}$$

APPROXIMATES.

In subtracting approximates, neglect the right-hand figure of the remainder, as uncertain; but an unit borrowed on the right, must be repaid, as in Ex. 1. 2. and 3. following.

Ex. 1.

$$\begin{array}{r}
 \text{From } 783.0625 \\
 \text{Sub. } 495.28571+
 \end{array}$$

Rem. 287.7767 certain.

Ex. 2.

$$\begin{array}{r}
 \text{From } 549.4643- \\
 \text{Sub. } 78.0875
 \end{array}$$

Rem. 471.376 certain.

Ex. 3.

$$\begin{array}{r}
 \text{From } 12\frac{1}{19} = 12.05263+ \\
 \text{Sub. } 4\frac{2}{19} = 4.10526+ \\
 \hline
 \text{Rem. } 7\frac{18}{19} = 7.9473 \text{ certain.}
 \end{array}$$

Ex. 4.

$$\begin{array}{r}
 \text{From } 17\frac{2}{17} = 17.11765- \\
 \text{Sub. } 8\frac{5}{17} = 8.29412- \\
 \hline
 \text{Rem. } 8\frac{14}{17} = 8.8235 \text{ certain.}
 \end{array}$$

RULE

RULE II. If one of the given decimals is a repeater, and the other a finite decimal, give the repeater one place more than the finite decimal, and in subtracting borrow 9 on the right hand.

But if both major and minor repeat, give them an equal number of places, and then subtract as above.

	Ex. 1.	Ex. 2.	Ex. 3.
From	.7145833	.525	.9989583
Sub.	.634375	.3333	.0291666
Rem.	.0802083	.1916	.9697916

In Ex. 1. and 2. I give the repeater one place more than the finite decimal, and by this means I obtain the repeating figure of the remainder. But in Ex. 3. I give the two repeaters an equal number of places.

In Ex. 2. and 3. I borrow 9 on the right hand.

It may be a proper exercise for the learner to reduce back the decimals that occur in these three examples to the vulgar fractions from which they result; then subtract the vulgar fractions, and reduce the remainders to decimals.

RULE III. If both the given decimals be circulates, make the circles conterminous, and work as in integers; only, if in the left-hand column of the circles, you foresee, that, in subtracting the figure of the minor from that of the major, one must be borrowed, in this case add 1 to the right-hand figure of the minor, and then subtract.

If one of the given decimals be a circulate, and the other a repeater, give the repeater the form of a conterminous circulate, and then subtract as above.

If one of the given decimals be a circulate, and the other a finite decimal, extend the finite part of the circulate to as many places as there are figures in the finite decimal, and then subtract.

EXAMPLE I.

$$\text{From } \frac{25}{27} = .925,925, = .925925,$$

$$\text{Sub. } \frac{4}{7} = .571428, = .571428,$$

$$\text{Rem. } .354497,$$

EXAMPLE II.

$$\text{From } \frac{9}{14} = .6428571, = .64,285714,$$

$$\text{Sub. } \frac{5}{28} = .17,857142, = .17,857142,$$

$$\text{Rem. } .46,428571,$$

In this last example, because, in the left-hand column of the circles, 8 cannot be subtracted from 2 without borrowing; therefore I add 1 to the right-hand figure of the minor, and say, $1 + 2 = 3$, and 3 from 4, and 1 remains. The reason is obvious: for, supposing the circles reiterated on the right of the example, it would be, 8 from 2 I cannot, but 8 from 12, and 4 remains; 1 borrowed, and 2, make 3, &c.

EXAMPLE III.

$$\text{From } \frac{13}{14} = .9,285714, = .9,285714,$$

$$\text{Sub. } \frac{2}{3} = .\bar{6} = .\bar{6},666666,$$

$$\text{Rem. } .2,619047,$$

EXAMPLE IV.

$$\text{From } 18\frac{1}{12} = 18.08\bar{3} = 18.08,333333,$$

$$\text{Sub. } 4\frac{5}{14} = 4.3,571428, = 4.35,714285,$$

$$\text{Rem. } 13.72,619047,$$

In the above two examples I give the repeaters the form of conterminous circulates.

EXAMPLE V.

$$\text{From } \frac{5}{13} = .384615, = .384,615384,$$

$$\text{Sub. } \frac{1}{8} = .125 = .125$$

$$\text{Rem. } .259,615384,$$

EX.

EXAMPLE VI.

$$\begin{array}{r} \text{From } \frac{3}{4} = .75 \\ \text{Sub. } \frac{1}{14} = .0714285, = .07142857, \\ \hline \text{Rem. } .6785714-, \end{array}$$

In the last two examples I extend the finite part of the circulate to as many places as there are figures in the finite decimal, by which means like things come to be subtracted, and I obtain the exact circle of the remainder.

CHAP. V.

MULTIPLICATION OF DECIMALS.

In multiplication and division there may happen nine varieties, arising from the different nature of the numbers that may occur in the operation; and these are of three sorts, *viz.* integers, mixt numbers, and pure decimals.

Now, since the multiplier or divisor may be of three kinds, and the multiplicand or dividend of as many, there must of consequence be nine varieties; which are these following.

- | | |
|---------------------------------------|--|
| An integer may multiply or divide | { an integer,
a mixt number,
a pure decimal. |
| A mixt number may multiply or divide | { an integer,
a mixt number,
a pure decimal. |
| A pure decimal may multiply or divide | { an integer,
a mixt number,
a pure decimal. |

Of these varieties, the first belongs properly to vulgar arithmetic, the other eight occur in decimal operations.

But in multiplication and division of decimals, there will occur other nine varieties, arising likewise from the nature of the numbers; which may either be finite, repeating, or circulating.

And

And since the multiplier or divisor may be of three sorts, and the multiplicand or dividend of as many, there must of course be nine varieties; and these are so obvious, that it would be losing time here to enumerate them.

Before I enter on multiplication, I shall lay down a rule for pointing the product, which is of a general nature, and extends to decimals of every sort, whether finite, repeating, or circulating; and is as follows.

GENERAL RULE.

Give so many decimal places to the product, on the right, as are in both factors; and if the product has not so many figures, supply that defect by prefixing ciphers.

We now proceed to multiplication.

RULE I. If both factors are finite or approximate, work exactly as in multiplication of integers.

Ex. 1.

$$\begin{array}{r} .785 \\ .75 \\ \hline \end{array}$$

$$\begin{array}{r} 3925 \\ 5495 \\ \hline .58875 \end{array}$$

Ex. 2.

$$\begin{array}{r} .125 \\ .25 \\ \hline \end{array}$$

$$\begin{array}{r} 625 \\ 250 \\ \hline .03125 \end{array}$$

Ex. 3.

$$\begin{array}{r} .0375 \\ .05 \\ \hline \end{array}$$

$$\begin{array}{r} .001875. \end{array}$$

Ex. 4.

$$\begin{array}{r} 6.875 \\ .25 \\ \hline \end{array}$$

$$\begin{array}{r} 34375 \\ 13750 \\ \hline 1.71875 \end{array}$$

Ex. 5.

$$\begin{array}{r} .8765 \\ .12.5 \\ \hline \end{array}$$

$$\begin{array}{r} 43825 \\ 105180 \\ \hline 10.95625 \end{array}$$

Ex. 6.

$$\begin{array}{r} 5.25 \\ 3.5 \\ \hline \end{array}$$

$$\begin{array}{r} 2625 \\ 1575 \\ \hline 18.375 \end{array}$$

Ex.

Ex. 7.	Ex. 8.	Ex. 9.	Ex. 10.
$\begin{array}{r} 85 \\ .5 \\ \hline 42.5 \end{array}$	$\begin{array}{r} .95 \\ 7 \\ \hline 6.65 \end{array}$	$\begin{array}{r} 4.75 \\ .6 \\ \hline 28.50 \end{array}$	$\begin{array}{r} 73 \\ 2.5 \\ \hline 365 \\ 146 \\ \hline 182.5 \end{array}$

In Ex. 2. and 3. the product not affording so many decimal places as are in the multiplicand and multiplier, I supply the defect by prefixing ciphers.

The reason of giving as many decimal places to the product as are in both factors, appears by considering that the operation is the same here as in multiplication of vulgar fractions. Thus, $.785 \times .75 = \frac{785}{1000} \times \frac{75}{100} = \frac{58875}{100000} = .58875$. And thus, $6.875 \times .25 = \frac{6875}{1000} \times \frac{25}{100} = \frac{171875}{100000} = 1.71875$.

To multiply by 10, 100, 1000, &c. move the decimal point so many places toward the right hand as there are ciphers in the multiplier.

Thus :

And thus :

$.4375 \times 10 = 4.375$	$6.875 \times 10 = 68.75$
$.4375 \times 100 = 43.75$	$6.875 \times 100 = 687.5$
$.4375 \times 1000 = 437.5$	$6.875 \times 1000 = 6875.$
$.4375 \times 10000 = 4375.$	$6.875 \times 10000 = 68750.$

APPROXIMATES.

In multiplying approximates the certain places of the product may be determined by one or other of the two rules following, *viz.*

1. If both factors are approximates, the uncertain places of the product will be one more than the number of places in the longest factor.

2. If one of the factors be finite, and the other approximate, the uncertain places of the product will be one more than the number of places in the finite factor.

Ex. 1.

$$\begin{array}{r}
 245.118- \\
 .3529+ \\
 \hline
 2206062 \\
 490236 \\
 1225590 \\
 735354 \\
 \hline
 86.5021422
 \end{array}$$

Ex. 2.

$$\begin{array}{r}
 .210526+ \\
 2.875- \\
 \hline
 1052630 \\
 1473682 \\
 1684208 \\
 421052 \\
 \hline
 .605262250
 \end{array}$$

In Ex. 1. the integral part of the product, *viz.* 86, is certain, and all the decimal places on the right are uncertain. In Ex. 2. only four places on the left, *viz.* .6052, are certain, and all the other places uncertain.

The reason of Rule 1. is plain. For if in Ex. 1. we make the longest factor the multiplier, and the total product will be the same either way, it is obvious, that in this case we shall have six particular products, in each of which the right-hand figure will be uncertain, and consequently we shall have six uncertain places in the total product toward the right, and also one uncertain place more on account of the uncertain carriage from the column in which the right-hand figure of the last particular product stands.

The reason of Rule 2. is also obvious. For in Ex 2. by making the finite factor the multiplier, we have four particular products, in each of which the right-hand figure is uncertain; and so we have four uncertain places in the total product, and one uncertain place more arising from the uncertain carriage.

The carriage in some cases may affect several columns on the left, and thereby render so many more figures uncertain.

The surest way therefore to determine the certain places in the product of approximates, is by a second operation, giving the approximates contrary signs; for then, so far as the two products agree, the figures are certain. The second operations of the two former examples follow.

Ex. 1.

Ex. 1.

245.117+
 .353—

735351
 1225585
 735351
 86.526301

Ex. 2.

.210527—
 2.875—

1052635
 1473689
 1684216
 421054
 .605265125

In Ex. 1. 86.5 is certain, and all the other figures uncertain. In Ex. 2. .60526 are the only certain places.

Because the multiplication of decimals that consist of many places, proves, in the way hitherto practised, a tedious operation, I shall here explain a method whereby decimals of this sort, whether finite or approximate, may be multiplied expeditiously, and at the same time have the decimal places in the product limited to any number proposed. This may be effected by the following

R U L E.

Under the multiplicand place the multiplier inverted, so that its units place may stand under that place of the multiplicand to which you propose to limit the product; then multiply the right-hand figure of the multiplier into that figure of the multiplicand which stands directly over it, taking in the carriage from the right, and go on to multiply it into all the other figures on the left. Proceed in like manner with every other figure of the multiplier, placing the right-hand figures of all the particular products directly under other. The total product will be approximate, and the right-hand figure uncertain.

To make this rule more easily understood, the reader may look back to the multiplication of integers; where it was observed, that instead of beginning with the right-hand figure of the multiplier, we may begin with the left, and still have a just product, provided the right-hand figure of every particular product be placed di-

rectly under the multiplying figure. Now, the working an example, both in this manner, and also by the rule, and comparing the steps and results of the two operations, will throw a light upon the matter, and unfold the reason of the rule. Which take as follows.

Multiply 18,634375 into 9.875, and limit the product to four decimal places.

By the rule.

18.634375
578.9

1677093

149075

13044

931

184.0143+

By the other method,

18.634375
A 9.875

1677093|75

149075|000

13044|0625

931|71875

184.0144|53125

B

In working by the rule, I invert the multiplier, and place 9, the units figure, under 3, the fourth place of decimals, because the product is limited to four decimal places; then I multiply, saying, $9 \times 3 = 27$, and 6 carried from the right makes 33, &c. In multiplying by 8, I say, $8 \times 4 = 32$, and 3 of carriage makes 35; so I set 5 under 3; and proceed in like manner to multiply the figures on the left. The right-hand figure of the product is defective, as wanting the carriage from the columns cut off on the right by the line A B. The figures expressing the sum of the columns so cut off, are so many uncertain places of the product, when the factors are approximate, and on that account to be rejected as useless. The figures, moreover, on the right of the line A B, show how far the operation is contracted, or how much labour is saved in working by the rule.

In working by this rule, some teach to carry from the right hand 1 for every product betwixt 5 and 15, and 2 for every product betwixt 15 and 25, &c.; but rather than adopt a method so irregular, and at the same time

time inaccurate, it is better to allow the product one decimal place more than properly you have occasion for.

If there be no units in the multiplier, in this case set the right-hand figure of the inverted multiplier under that figure of the multiplicand, below which it would have stood had there been units.

Ex. Multiply .825 by .825, limiting the product to three decimal places.

By the rule. The common way at large.

.825	.825
528.	.825
660	4125
16	1650
4	6600
.680+	.680625

The decimal places of the factors may either be retained at full length, or turned into approximates before you begin to multiply.

Ex. Multiply 25.849013625 by 42.97235, limiting the product to two decimal places.

$$\begin{array}{r}
 25.849013625 \\
 \times 42.97235 \\
 \hline
 103396 \\
 5169 \\
 2326 \\
 180 \\
 5 \\
 \hline
 1110.76+
 \end{array}$$

I shall next turn the decimals of the former example into approximates, and then the operation will be as follows.

By

By the rule.

Common way at large.

$$\begin{array}{r} 25.85- \\ +79.24 \\ \hline 103400 \\ 5170 \\ 2326 \\ 180 \\ \hline \end{array}$$

Prod. 1110.76+

$$\begin{array}{r} 25.85- \\ 42.97+ \\ \hline 18095 \\ 23265 \\ 5170 \\ 10340 \\ \hline \end{array}$$

1110.7745

It remains to be observed, that the want of carriage from the right hand may sometimes affect more columns on the left than one, and thereby occasion more uncertain figures in the product than that on the right hand. The best security on this head is, never to limit the product to fewer than four or five decimal places.

To conclude, when decimals to be multiplied are long, you may frequently perform the operation more easily in vulgar fractions, and then reduce the product to a decimal.

RULE II. If the multiplier be finite, and the multiplicand repeat, in multiplying carry at 9 on the right hand; and before you add, prolong the repetends of the particular products, till their right-hand figures stand directly under one another; and in adding, carry at 9 on the right hand.

The product repeats, as in Ex. 1. 2. &c.; or turns out finite, as in Ex. 6.

Ex. 1.

$$\begin{array}{r} .3 \\ 4 \\ \hline 1.3 \end{array}$$

Ex. 2.

$$\begin{array}{r} .1\bar{6} \\ 7 \\ \hline 1.1\bar{6} \end{array}$$

Ex. 3.

$$\begin{array}{r} 27.08\bar{3} \\ .5 \\ \hline 13.541\bar{6} \end{array}$$

Ex. 4.

$$\begin{array}{r} 354.2\bar{6} \\ .0\bar{8} \\ \hline 28.341\bar{3} \end{array}$$

Ex. 5.

Ex. 5.

$$\begin{array}{r}
 4.0\phi \\
 5.2 \\
 \hline
 81\phi \\
 203\phi\phi \\
 \hline
 21.14\phi
 \end{array}$$

Ex. 6.

$$\begin{array}{r}
 6.4\phi \\
 123 \\
 \hline
 1930 \\
 128\phi\phi \\
 \hline
 64\phi\phi\phi \\
 \hline
 791.30
 \end{array}$$

Or thus:

$$\begin{array}{r}
 6.4\phi \\
 123 \\
 \hline
 1930 \\
 77200 \\
 \hline
 791.30
 \end{array}$$

Ex. 7.

$$\begin{array}{r}
 27.88\phi \\
 8.75 \\
 \hline
 13941\phi \\
 19518\phi\phi \\
 22306\phi\phi\phi \\
 \hline
 243.979\phi
 \end{array}$$

Ex. 8.

$$\begin{array}{r}
 538.2\phi \\
 .345 \\
 \hline
 26912\phi \\
 21529\phi\phi \\
 16147\phi\phi\phi \\
 \hline
 185.694\phi\phi
 \end{array}$$

Note. If the multiplier has ciphers on the right, instead of annexing ciphers to the product, reiterate its right-hand figure so many times as there are ciphers.

Ex. 1.

$$\begin{array}{r}
 79.\phi \\
 50 \\
 \hline
 3983.\phi
 \end{array}$$

Ex. 2.

$$\begin{array}{r}
 874.\phi \\
 900 \\
 \hline
 786900.0
 \end{array}$$

Ex. 3.

$$\begin{array}{r}
 84.\phi \\
 28000 \\
 \hline
 678\phi \\
 169\phi\phi
 \end{array}$$

$$\hline
 2373777.\phi$$

RULE III. If the multiplier be finite, and the multiplicand circulate, to the product of the right-hand figure of the circle add the carriage from the left, then proceed as in multiplication of integers; but before you add the particular products, make them conterminous, and then add as in addition of circulates.

The product commonly circulates; and then its circle is similar to the circle of the multiplicand, as in Ex. 1.

MULTIPLICATION of Part II.

2. 7. and 8.; but the product sometimes repeats, as in Ex. 5.; or it may turn out finite, as in Ex. 6.

Here it is obvious, that in multiplying $.481$ by 7, the carriage from the left would be 3; so I say, $7 \times 1 = 7$, and 3 of carriage, make 10, &c. The product circulates, and its circle $.370$, is similar to $.481$, the circle of the multiplicand.

Ex. 1.

$$\begin{array}{r} .481, \\ 7 \\ \hline \end{array}$$

3.370,

Ex. 2.

$$\begin{array}{r} 7.518, \\ .5 \\ \hline 3.7,592, \end{array}$$

Ex. 3.

$$\begin{array}{r} 7.518, \\ .05 \\ \hline .37,592, \end{array}$$

Ex. 4.

$$\begin{array}{r} 7.518, \\ .005 \\ \hline .037,592, \end{array}$$

In the above three examples the products are mixt circulates, the three figures on the right being the circle, and the figures on the left the finite parts.

Ex. 5.

$$\begin{array}{r} 8.962, \\ 24 \\ \hline 35,851, \\ 179,259, \\ \hline 215.111 \end{array}$$

Ex. 6.

$$\begin{array}{r} 4.7,857142, \\ 8.05 \\ \hline 239,285714, \\ 38285,714285, \\ \hline 38.524,999999, \\ \hline 1 \\ \hline 38.525 \end{array}$$

In Ex. 5. because the circle of the multiplicand consists of three places, the circle of every particular product will likewise consist of three places; and so, before I add, I make the circle of the second particular product conterminous with that of the first, by transferring its left-hand figure 9 to the right under 1; and in adding I take in the carriage from the left-hand column of the circles, and the product is a mixt repeater, equal to $215\frac{1}{9}$.

In Ex. 6. I transfer 85, in the second particular product, from the left to the right of the circle, which makes

makes it conterminous with the circle above it; and the circle of the product being a series of nines, I add 1 to the finite part, and the total product is finite.

The reason of adding the carriage from the left to the product of the right-hand figure of the circle, and of making the circles of the particular products conterminous, is the same here as in addition of circulates.

Ex. 7.

$$\begin{array}{r}
 46.02439, \\
 \underline{7.42} \\
 92,04878, \\
 1840,97560, \\
 \underline{32217,07317,} \\
 341.50,09756,
 \end{array}$$

Ex. 8.

$$\begin{array}{r}
 86.53,571428, \\
 \underline{58.75} \\
 43267,857142, \\
 605749,999999, \\
 \underline{6922857,142857,} \\
 43267857,142857, \\
 \underline{5083.9732,142857,}
 \end{array}$$

Note. If the multiplier has ciphers on the right, first point the product in the manner you would do were the ciphers subjoined to it; and then, instead of annexing the ciphers, transfer so many figures from left to right as serve to complete the circle.

Ex. 1.

$$\begin{array}{r}
 .72, \\
 \underline{20} \\
 14.54,
 \end{array}$$

Ex. 2.

$$\begin{array}{r}
 3.962, \\
 \underline{480} \\
 31,703, \\
 \underline{158,518,} \\
 1902.72
 \end{array}$$

Ex. 3.

$$\begin{array}{r}
 4.3,571428, \\
 \underline{600} \\
 2614.285714,
 \end{array}$$

In Ex. 2. the product repeats, and so there is no occasion to transfer any figures from left to right.

RULE IV. If the multiplier be interminate, reduce it to a vulgar fraction, as directed in reduction of decimals, Prob. V.; then multiply the given multiplicand by the numerator, (working as in integers, if the multiplicand be finite; or as directed in Rule 2. if it repeat; or as

prescribed in Rule 3. if it circulate); and divide the product by the denominator.

Here there are six cases; for the multiplier may repeat or circulate, and may multiply a finite, a repeating, or circulating multiplicand.

CASE I. When a repeating multiplier multiplies a finite multiplicand.

EXAMPLE I.

Multiply 638.25 by $.4 = \frac{4}{10}$

$$\begin{array}{r}
 638.25 \\
 \underline{4} \\
 9)2553.00 \text{ (283.6 product sought.} \\
 \underline{18} \\
 75 \\
 \underline{72} \\
 33 \\
 \underline{27} \\
 60 \\
 \underline{54} \\
 *6
 \end{array}$$

EXAMPLE II.

Multiply 872.35 by $.07 = \frac{7}{100} = \frac{7}{9 \times 10}$

Here I divide first by 9, and then I divide that quot by 10, which is done by moving the decimal point one place to the left.

$$\begin{array}{r}
 872.35 \\
 \underline{7} \\
 9)6106.45(\\
 \underline{54} \\
 67.8494 \text{ prod.}
 \end{array}$$

EX.

EXAMPLE III.

Multiply 7989 by $47.\dot{8} = \frac{428}{9}$

The numerator of the vulgar fraction is found by subtracting the finite part, as follows.

$$\begin{array}{r} 47.\dot{8} \\ 47 \\ \hline 428 \end{array}$$

$$\begin{array}{r} 7989 \\ 428 \\ \hline 63912 \\ 15978 \\ \hline 31956 \\ 9)3419292(\\ \hline 379921.\dot{8} \text{ prod.} \end{array}$$

EXAMPLE IV.

Multiply 87.34 by $2.6\dot{7} = \frac{241}{90} = \frac{241}{9 \times 10}$

The numerator is found as before.

$$\begin{array}{r} 2.6\dot{7} \\ 26 \\ \hline 241 \end{array}$$

I divide first by 9, and then by 10, as in Ex. 2.

$$\begin{array}{r} 87.34 \\ 241 \\ \hline 8734 \\ 34936 \\ \hline 17468 \\ 9)21048.94(\\ \hline 233.877\dot{x} \text{ product.} \end{array}$$

EXAMPLE V.

Multiply 680.748 by $3.21\dot{6} = \frac{2895}{900} = \frac{2895}{9 \times 100}$

$$\begin{array}{r} 3.21\dot{6} \\ 321 \\ \hline 2895 \text{ num.} \end{array}$$

After dividing by 9, I divide by 100, which is done by moving the decimal point two places to the left.

$$\begin{array}{r} 680.748 \\ 2895 \\ \hline 3403740 \\ 6126732 \\ \hline 5445984 \\ 1361496 \\ \hline 9)1970765.460(\\ \hline 2189.73940 \end{array}$$

116 MULTIPLICATION of Part II.

Note 1. When the multiplier is $.3$ or $.6$, it will be shorter and easier to work by the following directions.

1. When the multiplier is $.3$ take $\frac{1}{3}$ of the multiplicand.

Ex. 1.

Multiply 368 by $.3$

$$\begin{array}{r} 3)368(\\ \hline \text{Prod. } 122.6 \end{array}$$

Ex. 2.

Multiply 47.32 by $.03$

$$\begin{array}{r} 3)47.32(\\ \hline 1.5776 \text{ prod.} \end{array}$$

2. When the multiplier is $.6$ take $\frac{1}{3}$ of the multiplicand twice.

Ex. 1.

Multiply 12.49 by $.6$

$$\begin{array}{r} 3)12.49(\\ \hline 4.163 \\ 4.163 \\ \hline \text{Prod. } 8.326 \end{array}$$

Ex. 2.

Multiply 12.49 by $.06$

$$\begin{array}{r} 3)12.49(\\ \hline 4.163 \\ 4.163 \\ \hline .8326 \text{ prod.} \end{array}$$

Note 2. You may turn the multiplicand into multiplier, and then work as directed in Rule 2. Example 4. done in this way follows.

Multiply 87.34 by 2.67

$$\begin{array}{r} 2.67 \\ 87.34 \\ \hline 1071 \\ 8033 \\ 187444 \\ 2147222 \\ \hline \end{array}$$

Product 233.8778 the same as before.

Note 3. Because every repeating figure is so many ninth parts, we may multiply by the repeating figure as by an integer, and take $\frac{1}{9}$ of the product, placing the product of the next figure of the multiplier under the finite part of the quot, as in addition, and allowing no decimal

decimal place in the product for the repeating figure. The former example done in this manner follows.

Multiply 87.34 by 2.67

$$\begin{array}{r}
 87.34 \\
 2.67 \\
 \hline
 9)61138 \\
 \hline
 \underline{6793}x \\
 52404 \\
 17468 \\
 \hline
 \end{array}$$

Product 233.877x the same as formerly.

Here I place 7 beyond the right of the multiplicand, as we do ciphers in multiplication of integers. Under 6793, the finite part of the quot, I place 52404, as in addition. I give only three decimal places to the product for those in the two factors, and one more for the repeating x.

The product of the repeating 7 into the multiplicand may also be found by taking $\frac{1}{7}$ of the multiplicand, and multiplying the quot by 7, as in the margin.

$$\begin{array}{r}
 9)87.34 \\
 \hline
 9.704 \\
 7 \\
 \hline
 67.93x
 \end{array}$$

The same ways of operation may be used in Cases 2. and 3. following, but the method prescribed in the rule is more simple and easy.

CASE II. When both factors repeat.

EX.

EXAMPLE I.

Multiply $6.8\bar{z}$ by $.7 = \frac{7}{9}$

In dividing by 9, after the dividend is exhausted, I continue the division by annexing to the remainders the repeating figure of the dividend; and the quot or product sought comes out a mixt circulate.

$$\begin{array}{r}
 6.8\bar{z} \\
 \underline{7} \\
 9)47.8\bar{z} (5.3,148, \text{ prod.} \\
 \underline{45} \quad \cdot \\
 28 \\
 \underline{27} \\
 *13 \\
 \underline{9} \\
 43 \\
 \underline{36} \\
 73 \\
 \underline{72} \\
 *1
 \end{array}$$

EXAMPLE II.

Multiply 5.6 by $2.3 = \frac{21}{9}$

$$\begin{array}{r}
 2.3 \\
 \underline{2} \\
 21 \text{ numerator.} \\
 5.6 \\
 \underline{21} \\
 1183 \\
 9)119.0(\\
 \underline{13.7} \text{ prod.}
 \end{array}$$

EX.

EXAMPLE III.

Multiply $1.\dot{x}$ by $1.\dot{x} = \frac{10}{9}$

$$\begin{array}{r} 1.\dot{x} \\ 1 \\ \hline \end{array}$$

10 num.

The multiplication by 10 is performed by moving the decimal point one place to the right.

$$\begin{array}{r} 1.\dot{x} \\ 10 \\ \hline \end{array}$$

$$9)11.\dot{x}111111111($$

1.234567901,2 prod.

EXAMPLE IV.

Multiply $16.\dot{8}$ by $3.8\dot{4} = \frac{346}{90} = \frac{346}{9 \times 10}$

$$\begin{array}{r} 3.8\dot{4} \\ 38 \\ \hline \end{array}$$

346 num.

$$\begin{array}{r} 16.\dot{8} \\ 346 \\ \hline \end{array}$$

$$\begin{array}{r} 1013 \\ 6785 \\ 5066 \\ \hline \end{array}$$

After dividing by 9, I divide by 10, which is done by moving the decimal point one place to the left.

$$9|0)5843.8555555555($$

64.9,283950617,2 prod.

Note. When the multiplier is $.3$ or $.6$, it will be easier to work by the directions of Note 1. Case 1.

Ex. 1.

Multiply $18.\dot{6}$ by $.3$

$$3)18.\dot{6}($$

6.7 product

Ex. 2.

Multiply $18.\dot{6}$ by $.03$

$$3)18.\dot{6}($$

.67 product

Ex.

Ex. 3.

Multiply $489.\dot{8}$ by $\dot{6}$

$$\begin{array}{r} 3)489.\dot{8}55(\\ \underline{163.185,} \\ 163.185, \\ \hline 326.370, \text{ product} \end{array}$$

Ex. 4.

Multiply $489.\dot{8}$ by $\dot{06}$

$$\begin{array}{r} 3)489.\dot{8}55(\\ \underline{163.185,} \\ 163.185, \\ \hline 32.6,370, \text{ product} \end{array}$$

CASE III. When a repeating multiplier multiplies a circulating multiplicand.

EXAMPLE I.

Multiply 24.36 , by $\dot{4} = \frac{4}{9}$

In multiplying by 4 , I take in the carriage from the left of the circle; and in dividing by 9 I continue the division by annexing to the remainders the circulating figures of the dividend.

$$\begin{array}{r} 24.36, \\ \underline{4} \\ 9)97.45.(10.82, \text{ prod.} \\ \underline{9} \\ *74 \\ \underline{72} \\ 25 \\ \underline{18} \\ *74 \end{array}$$

EXAMPLE II.

Multiply 38.428571 , by $\dot{07} = \frac{7}{90} = \frac{7}{9 \times 10}$

After dividing by 9 , I divide again by 10 , which is done by moving the decimal point one place to the left.

$$\begin{array}{r} 38.428571, \\ \underline{7} \\ 90)268.999999, \\ \hline 2.98 \text{ prod.} \end{array}$$

EX-

EXAMPLE III.

Multiply 555.23,12195, by $45.\bar{8} = \frac{410}{9} = \frac{41 \times 10}{9}$

45. $\bar{8}$ 45

410 num.

555.23,12195,

410

55523,12195,

2220924,87804,

9)227644.7,99999,

25293.86

prod.

After multiplying by 41,
I again multiply by 10;
which is done by giving the
product only six decimal
places, instead of seven.

EXAMPLE IV.

Multiply 37.53658, by $2.4\bar{7} = \frac{223}{90}$

2.4 $\bar{7}$ 24

223 num.

37.53658,

223

112,60975,

750,73170,

7507,31707,

9)8370.65853,

93.0,07317, product.

EXAMPLE V.

Multiply 78.53,571428, by $54.8\bar{3} = \frac{4235}{90}$

54.8 $\bar{3}$ 548

4935 num.

78.53,571428,

4935

39267,857142,

235607,142857,

7068214,285714,

31414285,714285,

9)387573.74,999999,(

4306.374,999999,

I

Prod. 4306.375

finite

Note. When the multiplier is .3 or .6 it will be easier to work by the directions of Note 1. Case 1.

Ex. 1

Multiply .851, by .3

$$\begin{array}{r} 3).851,851,851,851,(\\ \hline \end{array}$$

Prod. .283950617,28 &c.

Ex. 2.

Multiply .846153, by .03

$$\begin{array}{r} 3).846153,(\\ \hline \end{array}$$

Prod. .0,282051,

Ex. 3.

Multiply .53,571428, by .6

$$\begin{array}{r} 3).53,571428,(\\ \hline \end{array}$$

.17,857142,

.17,857142,

Prod. .35,714285,

Ex. 4.

Multiply .461538, by .06

$$\begin{array}{r} 3).461538,(\\ \hline \end{array}$$

.153846,

.153846,

Prod. .0,307692,

In x. 1. the given circulate .851, = $\frac{23}{27}$, being no multiple of the divisor 3, the circle of the quot runs on to nine places. See the remarks subjoined to the specimen of pure circulates in reduction of decimals, Prob. I.

CASE IV. When a circulate multiplies a finite multiplicand.

EXAMPLE I.

Multiply 825 by .36, = $\frac{36}{99}$

825

36

4950

2475

99)29700(300 product

297

EXAMPLE II.

Multiply 56.874 by 4.296, = $\frac{4296}{999}$

56.874

$$\begin{array}{r}
 56.874 \\
 \underline{4292} \\
 113748 \\
 511866 \\
 \underline{113748} \\
 227496 \\
 \underline{999)244103.208(244.3475} \\
 1998 \dots \dots \\
 \underline{4430} \\
 3996 \\
 \underline{4343} \\
 3996 \\
 \underline{3472} \\
 2997 \\
 \underline{4750} \\
 3996 \\
 \underline{7548} \\
 6993 \\
 \underline{5550} \\
 4995 \\
 \underline{*555}
 \end{array}$$

Instead of dividing by 999, as above, it will be shorter and easier to divide first by 1000, which is done by moving the decimal point three places to the left, and then proceed by the method of dividing by 9, 99, 999, &c. taught in division of integers, as in the margin.

$$\begin{array}{r}
 999)244.103208 \\
 .244103 \\
 \underline{244} \\
 244.347555
 \end{array}$$

EXAMPLE III.

Multiply 458.75 by 8.481, = $\frac{8413}{999}$

Q 2

458.75

$$\begin{array}{r}
 458.75 \\
 8473 \\
 \hline
 137625 \\
 321125 \\
 183500 \\
 367000 \\
 \hline
 999)3886988.75(3890.87,962, \\
 2997 \dots \dots \\
 \hline
 8899 \\
 7992 \\
 \hline
 9078 \\
 8991 \\
 \hline
 8787 \\
 7992 \\
 \hline
 7955 \\
 6993 \\
 \hline
 *9620 \\
 8991 \\
 \hline
 6290 \\
 5994 \\
 \hline
 2960 \\
 1998 \\
 \hline
 *962
 \end{array}$$

But the other method of division is shorter and better; only observe, that the circle of the product will be similar to that of the multiplier, or it will consist of so many places as there are 9's in the divisor; and in order to bring it out complete, you must to the sum of the right-hand column add the carriage from the left of the circle. The former product, divided in this manner, follows.

Here

Here I first divide by 1000, by moving the decimal point three places to the left; and in adding, I add 1, the carriage from the left of the circle, to the right-hand column.

$$\begin{array}{r} 999) 3886.98,875, \\ \underline{3.88,698,} \\ 388, \\ \underline{3890.87,962,} \end{array}$$

If you want to bring out a repetition of the circle, it may be done by continuing the second partial quot to its full extent, and then proceeding in the other parts of the operation as before. The former example, continued to a repetition of the circle, follows.

By continuing the third partial quot to its full extent, the circle will come out thrice; and by continuing the fourth, it will come out four times, &c.

$$\begin{array}{r} 3886.98,875, \\ 3.88,698,875, \\ 388,698, \\ 388, \\ \underline{3890.87,962,962,} \end{array}$$

If at any time, instead of a circle, you have a series of nines, in this case 1 must be added to the finite part, and the quot or total product will be finite.

Thus, the first product in Example 1. when divided in this manner by 99, will stand as in the margin.

$$\begin{array}{r} 99) 297.00 \\ \underline{2.97} \\ .2 \\ \underline{299.99,} \\ 1 \\ \underline{300} \end{array}$$

EXAMPLE IV.

Multiply 64.35 by .772, = $\frac{765}{990} = \frac{765}{99 \times 10}$

64.35

64.35

765

32175

38610

45045

99|0)49227.75,

492.27,

4.92,

4,

49724.99,

I

Prod. 49.725 finite.

Note. You may turn the multiplicand into multiplier, and then work as directed in Rule 3. The above example done in this manner follows.

.7,72,

64.35

38,63,

231,81,

3090,90,

46363,63,

49.724,99,

I

Prod. 49.725 the same as before.

CASE V. When a circulate multiplies a repeating multiplicand.

EXAMPLE I.

Multiply 8.02083 by .72, = $\frac{72}{99}$

8.02083

8.02083

72

1604168

56145833

Or thus :

99) 577.50000(5.83

495 ..

825792

330

297

*33

99) 5.7.75,

57,

5.833,

In the second way of division, the two places on the right hand are considered as the places of the circle; so I bring 1 of carriage from left to right, and the quot or total product repeats.

EXAMPLE II.

Multiply 18.783416 by 4.36, = $\frac{432}{99}$.

18.783416

432

37566833

563502500

7513366666

99) 8114.436000(81.964

792 ..

194

99

954

891

633

594

396

396

Or thus : 99) 81.144,36

811,44

8,11

8

81.963,99,

1

Prod. 81.964 finite.

EX.

EXAMPLE III.

Multiply 198.7052083 by .045, = $\frac{45}{990}$

198.7052083

45

9935260416

7948208333

990)8941.7343750(9.0320549,24,

891.....

317

297

203

198

543

495

487

396

915

891

*240

198

420

396

*24

Or thus :

8.9417343,75,

894173,43,

8941,73,

89,41,

89,

9.0320549,24,

CASE VI. When both factors circulate.

EXAMPLE I.

Multiply .714285, by .36, = $\frac{36}{99}$

.714285,

$$\begin{array}{r}
 .714285, \\
 \underline{36} \\
 4,285714, \\
 21,428571, \\
 \hline
 99)25,714285,7 \\
 \underline{257142,8} \\
 2571,4 \\
 \underline{25,7}
 \end{array}$$

$$\text{Prod. } .25,974025, = .259740,25$$

The circle of the first product is always similar to that of the multiplicand, and in the above example consists of six places; but to secure the carriage from right to left, and thereby complete the circle of the quot or total product, I transfer 7, the left-hand figure of the circle of the first product, to the right, and fill up the places under it with the figures that come in course, and from the sum of these figures on the right I carry 2, which completes the circle of the total product.

EXAMPLE II.

$$\begin{array}{r}
 \text{Multiply } .32,142857, \\
 \text{by } .6,81, \\
 \underline{6}
 \end{array}$$

$$\begin{array}{r}
 \text{Multiplier } \underline{675} \\
 160,714285, \\
 2249,999999, \\
 19285,714285, \\
 \hline
 99|0)21696,428571,4 \\
 \underline{216,964285,7} \\
 2,169642,8 \\
 \underline{21696,4} \\
 216,9 \\
 \underline{2,1}
 \end{array}$$

$$\text{Prod. } .21915,584415, = .219,155844,15$$

EXAMPLE III,

Multiply 53.571428,
by 2.18,

2

Multiplier 216

321,428571,

535,714285,

10714,285714,

99) 11571,428571,4

115,714285,7

1,157142,8

11571,4

115,7

1,1

Prod. 116.88,311688, = 1,16.8831,1688

From the examples hitherto adduced, it is obvious, that when one, two, or more figures in the right of the circle are the same with the like number in the right of the finite part, or integral part, of the product, these latter figures may be made part of the circle.

When both factors are circulates, the number of places in the circle of the total product will always be equal to the number of units in the least multiple of the two numbers that denote the number of places in the circles of the multiplicand and multiplier.

If therefore the circle of the first product contain fewer places than there are units in the said least multiple, you must, before you divide, prolong the circle of the first product till it have the number of places required, and one or two places more to secure the carriage from the right; as is done in the following example.

EX.

EXAMPLE IV.

Multiply 78.45,
by 8.296,
8

Multiplier 8288

627.63,
6276.36,
15690.90,
627636.36,

999)650231,272727,27

650,231272,72

650231,27

650,23

65,

650.882,154882, = 650.882154,882

In the above example, the circle of the multiplicand consists of two places, and the circle of the multiplier of three places, and the least multiple of 2 and 3 is 6; so I conclude, that the circle of the total product will consist of six places: but the circle of the first product being similar to that of the multiplicand, has only two places; so I prolong it to six places, and give it two places more in order to secure the carriage.

If the circle of the first product repeat, extend the repeating figure at pleasure; and proceed to find the circle of the total product as before.

EXAMPLE V.

$$\begin{array}{r}
 \text{Multiply } 6.851, \\
 \text{by } .72, \\
 \hline
 13,703, \\
 479,629, \\
 \hline
 99)498,3333333 \\
 4,9333333 \\
 493333 \\
 4933 \\
 49 \\
 \hline
 \text{Product } 4.98316,498
 \end{array}$$

If you foresee that the circle of the total product will run on to many places ; to save the trouble of a tedious addition, you may proceed thus : Continue the circle of the first product at pleasure ; divide it into periods, so as each period may consist of as many places as there are figures in the circle of the multiplier ; then add the first period to the second, and the result to the third period, and this result again to the fourth, and so on till you bring out the circle of the total product, or as many places of it as you think proper. Place the carriage-figure from each period under the right-hand figure of the period on the left ; add these carriage-figures to the results under which they stand ; and their sum will be the total product complete, or so many true figures of it. But here observe, that the results, before they be added to the subsequent periods, must be increased by having these carriage-figures added to them.

EXAMPLE VI.

Multiply 6.142857,
by .56097,

42,999999,
552,857142,
36857,142857,
307142,857142,

99999	3.4459	5,8571	42,857	142,85
		34459	93031	35888
		93030	35888	50173
		I		I

Tot.pr. 3.4459 93031 35888 50174+

First pr. 71428 | 5,7142 | 85,714 | 285,71 continued.

50174	21602	78745	64459
21602	78744	64459	93030
	I		I

Tot.pr. 21602 78745 6,4459 93031

The total product complete is

3.445993031358885017421602787456,4459 &c.

In the above example, the circle of the multiplicand consists of six places, and the circle of the multiplier of five places, and the least multiple of 5 and 6 is 30; and so the circle of the total product runs on to thirty places, as above. The learner may divide the first product the other way; and by comparing the steps of the two operations, the truth and reason of the rule will appear.

PRACTICAL QUESTIONS.

1. What is the price of 868 gallons of rum, at 5 s. per gallon? *Ans.* 217 l.
2. What will 56086 yards of tape cost, at 3 farthings per yard? *Ans.* 175 l. 5 s. 4½ d.

3. What is the price of 6548 lb. of tea, at 6 s. 8 d. per lb? *Ans.* 2182 l. 13 s. 4 d.
4. What is the price of 276 C 2 Q. at 3 s. 8½ d. per C? *Ans.* 604 l. 5 s. 4¼ d.
5. What is the value of .9989583 l. Sterling? *Ans.* 19 s. 11¾ d.
6. What is the value of .99982638 lb. Troy? *Ans.* 11 oz. 19 dw. 23 gr.
7. What is the price of 648½ yards of cloth, at 13 s. 4 d. per yard? *Ans.* 432 l. 6 s. 8 d.
8. What is the price of 456½ yards, at 6 s. 2 d. per yard? *Ans.* 140 l. 15 s. 1 d.
9. What is the price of 2000 yards ribbon, at 4¼ d per yard? *Ans.* 35 l. 8 s. 4 d.
10. What is the value of .49944196,428571, C.? *Ans.* 1 Q. 27 lb. 15 oz.
11. What is the price of 15 C. 0 Q. 16 lb. at 2 l. 6 s. 8 d. per C. *Ans.* 35 l. 6 s. 8 d.
12. What will a yearly pension of 113½ l. amount to in 2 years 7 months, reckoning 13 months to the year? *Ans.* 288 l.
13. A capital or stock consists of 13 equal shares, whereof A has 1 share, B 3, C 4, and D 5. They propose to make a dividend of 390 l. profits, what is each man's share of the dividend?

Multiply .076923, = $\frac{1}{13}$ by 390, *Ans.* A's share 30
 and the product is A's share; and B's — 90
 this share multiplied by 3, by 4, and C's — 120
 by 5, gives the other shares. D's — 150

Proof 390

CHAP. VI.

DIVISION OF DECIMALS.

Before we enter on Division, it will be proper to observe, that there are two rules for pointing the quot, both

both which are general in their nature, and extend to decimals of every sort, whether terminate or interminate; but unwilling to perplex the learner with too many things at once, I shall at present lay down only one of these rules; and afterwards, when the rule now to be assigned appears to be sufficiently exemplified, I shall then bring the other rule upon the field.

I. G E N E R A L R U L E.

The decimal places in the divisor and quot together must always be equal in number to those of the dividend.

The five following practical directions will make the application of the general rule easy.

1. When the divisor and dividend have an equal number of decimal places, the quot comes out an integer; as in Ex. 2.

2. When the decimal places of the dividend are more than those of the divisor, the number of decimal places in the quot must be equal to the excess; as in Ex. 1. 4. and 8.

3. When the decimal places of the divisor are more than those of the dividend, annex ciphers to the dividend, so as to make them equal, and the quot, by direction 1. will be integers; as in Ex. 3. 5. and 7.

4. When, after division is finished, the quot has not so many figures, as, by the general rule, it ought to have decimal places, supply that defect by prefixing ciphers; as in Ex. 6.

5. If, after the dividend is exhausted, there be a remainder, annex a cipher, or ciphers, to the remainder, and continue the division till 0 remain, or till the quot repeat or circulate, or till you think proper to limit it; as in Ex. 9. 10. 11. and 12.

We now proceed to division.

RULE I. If the divisor and dividend are both finite or approximate, work exactly as in division of integers.

Ex.

3. What is the price of 6548 lb. of tea, at 6 s. 8 d. per lb? *Ans.* 2182 l. 13 s. 4 d.

4. What is the price of 276 C 2 Q. at 3 s. $8\frac{1}{2}$ d. per C? *Ans.* 604 l. 5 s. $4\frac{1}{4}$ d.

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6. What is the value of .99982638 lb. Troy? *Ans.* 11 oz. 19 dw. 23 gr.

7. What is the price of $648\frac{1}{2}$ yards of cloth, at 13 s. 4 d. per yard? *Ans.* 432 l. 6 s. 8 d.

8. What is the price of $456\frac{1}{2}$ yards, at 6 s. 2 d. per yard? *Ans.* 140 l. 15 s. 1 d.

9. What is the price of 2000 yards ribbon, at $4\frac{1}{4}$ d per yard? *Ans.* 35 l. 8 s. 4 d.

10. What is the value of .49944196,428571, C.? *Ans.* 1 Q. 27 lb. 15 oz.

11. What is the price of 15 C. 0 Q. 16 lb. at 2 l. 6 s. 8 d. per C. *Ans.* 35 l. 6 s. 8 d.

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by 5, gives the other shares. D's — 150

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3. When the decimal places of the divisor are more than those of the dividend, annex ciphers to the dividend, so as to make them equal, and the quot, by direction 1. will be integers; as in Ex. 3. 5. and 7.

4. When, after division is finished, the quot has not so many figures, as, by the general rule, it ought to have decimal places, supply that defect by prefixing ciphers; as in Ex. 6.

5. If, after the dividend is exhausted, there be a remainder, annex a cipher, or ciphers, to the remainder, and continue the division till o remain, or till the quot repeat or circulate, or till you think proper to limit it; as in Ex. 9. 10. 11. and 12.

We now proceed to division.

RULE I. If the divisor and dividend are both finite or approximate, work exactly as in division of integers.

Ex.

Ex. 1.

$$\begin{array}{r}
 .75) .58875 (.785 \\
 \underline{525} \\
 637 \\
 \underline{600} \\
 375 \\
 \underline{375} \\
 0
 \end{array}$$

Ex. 2.

$$\begin{array}{r}
 2.5) 182.5 (73 \\
 \underline{175} \\
 75 \\
 \underline{75} \\
 0
 \end{array}$$

In Ex. 1. A decimal divides a decimal; and because the dividend has five decimal places, and the divisor only two, I give three decimal places to the quot, according to Direction 2.

In Ex. 2. A mixt number divides a mixt number, and the divisor and dividend having an equal number of decimal places, the quot comes out an integer, according to Direction 1.

The reason of the rule for pointing the quot is obvious; for multiplication gives as many decimal places to the product as are in both factors; but the dividend is the product of the divisor and quot, and so has as many decimal places as are in both; consequently the decimal places in the divisor and quot together must be equal in number to those of the dividend.

Ex. 3.

$$\begin{array}{r}
 .85) 476 (\\
 .85) 476.00 (560 \\
 \underline{425} \\
 510 \\
 \underline{510} \\
 0
 \end{array}$$

Ex. 4.

$$\begin{array}{r}
 7) .875 (.125 \\
 \underline{7} \\
 17 \\
 \underline{14} \\
 35 \\
 \underline{35} \\
 0
 \end{array}$$

In Ex. 3. A decimal divides an integer; and the dividend having no decimal place, I annex two ciphers, because there are two decimal places in the divisor, and the quot comes out an integer, according to Direction 3.

In Ex. 4. an integer divides a decimal; and because the

the dividend has three decimal places, and the divisor none, I give the quot three; by Direction 2.

Ex. 5.

$$\begin{array}{r} .375 \overline{) 12.75} (\\ .375 \overline{) 12.750} (34 \\ \underline{1125} \\ 1500 \\ \underline{1500} \end{array}$$

Ex. 6.

$$\begin{array}{r} 2.5 \overline{) .22875} (.0915 \\ \underline{225} \\ 37 \\ \underline{25} \\ 125 \\ \underline{125} \end{array}$$

In Ex. 5. a decimal divides a mixt number; and the divisor having three decimal places, and the dividend but two, I supply that defect by annexing a cipher, and the quot comes out an integer, by Direction 3.

In Ex. 6. a mixt number divides a decimal; and because the dividend has four decimal places more than the divisor, and the quot, after the division is finished, has only three figures, I supply this defect by prefixing a cipher to it, according to Direction 4.

Ex. 7.

$$\begin{array}{r} 3.75 \overline{) 180} (\\ 3.75 \overline{) 180.00} (48 \\ \underline{1500} \\ 3000 \\ \underline{3000} \end{array}$$

Ex. 8.

$$\begin{array}{r} 38 \overline{) 243.2} (6.4 \\ \underline{228} \\ 152 \\ \underline{152} \end{array}$$

In Ex. 7. a mixt number divides an integer; and the dividend having no decimal places, I supply that defect by annexing two ciphers, the number of decimal places in the divisor, and the quot is an integer, by Direction 3.

In Ex. 8. an integer divides a mixt number; and the divisor having no decimal place, and the dividend only one, I give one to the quot, according to Direction 2.

Ex. 9.

$$\begin{array}{r}
 .8)29(36.23 \\
 \underline{24} \\
 50 \\
 \underline{48} \\
 20 \\
 \underline{16} \\
 40 \\
 \underline{40}
 \end{array}$$

Ex. 10.

$$\begin{array}{r}
 .018).0024(.13 \\
 \underline{18} \\
 60 \\
 \underline{54} \\
 *6
 \end{array}$$

In Ex. 9. a decimal divides an integer; and after the dividend is exhausted, I annex a cipher to the remainder, and continue the division till 0 remain, according to Direction 5.

In Ex. 10. a decimal divides a decimal; and after the dividend is exhausted, I annex a cipher to the remainder, and continue the division till I find the quot repeats.

Ex. 11.

$$\begin{array}{r}
 11)8(\\
 11)8.0(.72, \\
 \underline{77} \\
 30 \\
 \underline{22} \\
 *8
 \end{array}$$

Ex. 12.

$$\begin{array}{r}
 3.25)76.75(23.615+ \\
 \underline{650} \\
 1175 \\
 \underline{975} \\
 2000 \\
 \underline{1950} \\
 500 \\
 \underline{325} \\
 1750 \\
 \underline{1625} \\
 125
 \end{array}$$

In Ex. 11. an integer divides an integer; and the dividend being less than the divisor, I annex a cipher to it; again, after the dividend is exhausted, I annex a cipher

pher to the remainder, and continue the division till I find the quot circulates.

In Ex. 12. a mixt number divides a mixt number: and after the dividend is exhausted, by annexing ciphers to the remainder, I continue the division till the quot has three decimal places; and as there is still a remainder, it might be carried further; but three decimal places being in most cases sufficiently accurate, here I chuse to limit it; so the quot is approximate.

In division of decimals the place of the first figure of the quot may likewise be known from the first dividual, much after the same manner as in division of integers, by the following

II. GENERAL RULE.

The place of the first figure of the quot is the same with the place of that figure in the dividend which stands over the units of the first product.

Thus, in the example of integers in the margin, the figure 9, that stands over 5, the units of the product of 9×35 , is in the place of hundreds; and therefore 9, the first figure of the quot, is likewise hundreds; and so the quot is 917 integers.

$$\begin{array}{r} 35)32095(917 \\ \underline{315} \\ 59 \\ \underline{35} \\ 245 \\ \underline{245} \end{array}$$

To illustrate the rule I shall give decimal places to the dividend of the above example; and thereby exhibit the varieties that will occur in pointing the quot.

Variety 1.

$$\begin{array}{r} 35)3209.5(91.7 \\ \underline{315} \\ 59 \\ \underline{35} \\ 245 \\ \underline{245} \end{array}$$

Var. 2.

$$\begin{array}{r} 35)320.95(9.17 \\ \underline{315} \\ 59 \\ \underline{35} \\ 245 \\ \underline{245} \end{array}$$

S 2

Var.

Var. 3.

35)32.095(.917

315..

59

35

245

245*Var. 5.*

35)32095(.00917

315..

59

35

245

245*Var. 4.*

35)3.2095(.0917

315..

59

35

245

245*Var. 6.*

35).032095(.000917

315..

59

35

245

245

In all the above varieties, the figure 0 in the dividend stands over the units of the first product: and in Var. 1. the figure 0 is in the place of tens, and accordingly 9, the first figure of the quot, is tens; in Var. 2. the figure 0 is in the place of units, and so 9 is units; in Var. 3. the figure 0 is in the place of primes, and so 9 is primes, &c.

Here observe, that 9, the first significant figure of the quot, in all the above varieties, as well as in the varieties that follow, must always be considered, in multiplying the divisor, as an integer; and, in pointing the first product, no decimal place is to be allowed for it.

We shall now keep the dividend an integer, and give decimal places to the divisor.

Var. 1.

3.5)32095(9170

31.5..

59

35

245

245

0.0

Var. 2.

.35)32095(91700

3.15..

59

35

245

245

0.00

Var.

Var. 3.

$$\begin{array}{r} .035 \overline{) 32095(917000} \\ \underline{0.315} \end{array}$$

59

35

245

245

0.000

Var. 4.

$$\begin{array}{r} .0035 \overline{) 32095(9170000} \\ \underline{0.0315} \end{array}$$

59

35

245

245

0.0000

In Var. 1. I say, $9 \times 3.5 = 31.5$; and the unit 1 standing under the place of thousands, the figure 9 is also thousands; and as 0 at last remains, I annex a cipher for the decimal .5 in the divisor; then dividing, I get 0 to the quot; and because 9 stands in the place of thousands, the quot is wholly integers.

In Var. 2. the unit 3 stands under the place of ten-thousands, and so 9 is ten-thousands; and to the remainder 0 I annex .00, for the two decimal places in the divisor; then dividing, I get 00 to the quot; and because 9 stands in the place of ten-thousands, the quot continues to be wholly integers. The process is the same in Var. 3. and 4.

Lastly, we shall allow decimal places to both dividend and divisor.

Var. 1.

$$\begin{array}{r} 3.5 \overline{) 320.95(91.7} \\ \underline{31.5} \end{array}$$

59

35

245

245

Var. 2.

$$\begin{array}{r} 3.5 \overline{) 32.095(9.17} \\ \underline{31.5} \end{array}$$

59

35

245

245

Var.

Var 3.

 $.35).32005(.917$ $3.15..$ 59 35 245 245

Var. 4.

 $3.5).32095(.0917$ $31.5..$ 59 35 245 245

In Var. 1. the units of the first product stand under tens of the dividend; and so 9, the first figure of the quot, is tens. In Var. 2. the units of the first product stand under units of the dividend, and so 9 is units. In Var. 3. the units of the first product stand under primes, and so 9 is primes, &c.

To divide by 10, 100, 1000, &c. is to move the decimal point one place toward the left for every cipher in the divisor.

Thus,

 $10)768(76.8$ $100)768(7.68$ $1000)768(.768$ $10000)768(.0768$

And thus,

 $10)17.28(1.728$ $100)17.28(.1728$ $1000)17.28(.01728$ $10000)17.28(.001728$

A P P R O X I M A T E S.

In dividing approximates, the certain places of the quot may be determined by the following

R U L E.

Place the divisor under the first dividual, and the number of certain figures in the quot shall be one less than the number of places from the left of the divisor to the first + or -, whether in the divisor or in the dividend.

Ex. 1.

Ex. 1.

Dividend 1110.79286078—
 Divisor 42.9723+

Certain places five, where-
 of three are decimals.

Ex. 2.

Dividend 1110.7929—
 Divisor 25.8490136+

Certain places six, where-
 of four are decimals.

Ex. 3.

Dividend 4.675
 Divisor 34.823—

Certain places four,
 all decimals.

Ex. 4.

Dividend 765.84632+
 Divisor 237.5

Certain places seven, whereof
 six are decimals.

But here it is to be observed, that the uncertain carriage may, in some cases, affect several columns on the left, and thereby render more figures of the quot uncertain than the rule prescribes. The surest way, therefore, is, to make two operations with contrary signs, and then the figures in which the two quots agree are certain.

In order to make the reason of the rule appear, it will be necessary to work an example or two.

Ex. 1.

A

42.9723+) 1110.79286078— (25.849

859446....

2513468

2148615

3648536

3437784

2107520

1718892

3886287

3867507

18780

B

Here

Here we stop, no more places being certain. The reason is obvious; for the right-hand figure of the first product, *viz.* 6, is uncertain; and consequently all the figures under it, on the right of the line A B, will be so too; that is, the last remainder and new dividual are uncertain, and of course the figure that would go next to the quot.

Ex. 2.

A	
25.8490136+	1110.7929—(42.9723
1033960	544
76832	3560
51698	0272
25134	32880
23264	11224
1870	216560
1809	430952
60	7856080
51	6980272
90	8758080
77	5470408
13	3287672
B	

Here again we stop, no more places being certain: and the reason is plain; for the right-hand figure of the dividend, *viz.* 9, being uncertain, when the first product is subtracted from the dividual, the remainder 356 will be uncertain, and of course all the figures on the right of the line A B will be so likewise; that is, the last remainder and new dividual are uncertain, and so will the figure be that would thence arise to the quot. Hence the rule appears to be just.

From these two examples it appears, that all the figures on the right of the line A B are uncertain and useless; if therefore a way of working, without writing down these

these useless figures, can be found, we shall then have a method of dividing long decimals, whether finite or approximate, so as to contract the operation, and limit the product to any number of decimal places proposed. And this may be effected by observing the following

R U L E.

Write the product of the first quotient-figure under the dividend; and from the situation of the units place, consider how many figures of the dividend must be retained to give the quot the number of decimal places intended; cut off the other figures on the right, and also the figures corresponding to them on the right of the divisor; then subtract; esteem this and every following remainder a new dividual; and for each new dividual drop a figure on the right of the divisor; but in multiplying the quotient-figures into the divisor, take in the carriage from the right hand; as in the following examples.

E X A M P L E I.

Divide 95.432756463275 by 3.4637528; and limit the quot to four decimal places.

Contracted by the rule.

$$\begin{array}{r}
 3.46375\overline{)28}95.4327\overline{)56463275}(27.5518 \\
 \dots\dots 6.92750\overline{)56} \\
 \hline
 261577 \\
 242462 \\
 \hline
 19115 \\
 17318 \\
 \hline
 1797 \\
 1731 \\
 \hline
 66 \\
 34 \\
 \hline
 32 \\
 27 \\
 \hline
 (5)
 \end{array}$$

In the above example the units of the first product standing under the place of tens, the first figure of the quot is tens; and hence it is easy to foresee, that six figures of the dividend retained will give four decimal places to the quot; and accordingly I cut off all the other figures on the right of the dividend; I cut off likewise from the divisor two figures that correspond to them.

At every new dividual I drop or omit a figure on the right of the divisor, and mark the figure so dropped by setting a point under it; and in multiplying the quotient-figure 7 into the divisor, I say, 7 times 7 is 49, and 3 of carriage from the right, (arising from $7 \times 5 = 35$), makes 52; so I set down 2, and carry 5. The same method is observed in multiplying every other quotient-figure into the divisor.

The same Example at large.

A

3.4637528)95.4327 56463275(27.55183

6.92750 56.....

261577004

242462696

191143086

173187640

179554463

173187640

63668232

34637528

290307047

277100224

132068235

103912588

29055647

B

In

In working the same example at large, the line A B shows how far the operation is contracted, and how much labour is saved.

But here observe, that by the rule for approximates the certain places of the quot are no more than five, viz. 27.551. And therefore, in all operations of this kind, care should be taken to limit the quot to so many places certain; as is done in the following example.

E X A M P L E II.

Divide 87.0763264525 by 9.365407024; limiting the quot to four decimal places certain.

$$\begin{array}{r}
 9.36540|7024)87.07632|64525(9.2976 \\
 \text{.....} \quad 84.28866|3216 \\
 \hline
 278766 \\
 187308 \\
 \hline
 91458 \\
 84288 \\
 \hline
 7170 \\
 6555 \\
 \hline
 615 \\
 561 \\
 \hline
 54
 \end{array}$$

Here we put a stop to the operation; because, by the rule for approximates, the next figure of the quot would be uncertain.

I shall conclude division of finite decimals with two very useful problems.

P R O B I.

From a given multiplier to find a divisor that gives a quot equal to the product.

T 2

R U L E

DIVISION of R U L E.

Part II.

Divide an unit with ciphers annexed by the given multiplier, and the quot will be the divisor sought.

E X A M P L E.

What divisor will give a quot equal to the product of 125 into the dividend?

Given multiplier 125)1.000(.008 divisor sought.
1000

Now, if any number be divided by .008, and the the same number be multiplied by 125, the quot and product will be equal.

.008)7315.000(914375 quot.

72... ..

11
8
35
32
30
24
60
56
40
40

7315
125
36575
14630
7315
914375 product.

The reason is plain: for an unit contains the quot .008 just 125 times; and consequently .008 dividing any number will give a quot 125 times greater than the dividend; that is, the quot will be equal to the product of the dividend multiplied by 125.

P R O B. II.

From a given divisor to find a multiplier that gives a product equal to the quot.

R U L E.

R U L E.

Divide an unit with ciphers annexed by the given divisor, and the quot will be the multiplier sought.

E X A M P L E.

What multiplier will give a product equal to the quot arising from the same number divided by .008 ?

Given divisor .008)1.000(125 multiplier sought.

$$\begin{array}{r} 800 \\ \hline 200 \\ 160 \\ \hline 400 \\ 400 \\ \hline \end{array}$$

Now, if any number be multiplied by 125, and the same number be divided by .008, the product and quot will be equal; as appears in the example following.

$$\begin{array}{r} 785 \\ \times 125 \\ \hline 3925 \\ 15700 \\ 78500 \\ \hline 98125 \text{ product} \end{array}$$

$$\begin{array}{r} .008)785.000(98125 \text{ quot.} \\ 72 \\ \hline 65 \\ 64 \\ \hline 10 \\ 8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline \end{array}$$

RULE II. If a finite divisor divide a repeating dividend, work as in integers; but in continuing the division, instead of annexing ciphers to the remainder, annex the repeating figure of the dividend.

Ex.

Ex. 1.

$$\begin{array}{r} 5) 33.06 \\ \underline{30} \\ *3 \end{array}$$

Ex. 2.

$$\begin{array}{r} 4) 5.16(1.2916 \\ \underline{4} \\ 11 \\ \underline{8} \\ 36 \\ \underline{36} \\ 6 \\ \underline{4} \\ 26 \\ \underline{24} \\ *2 \end{array}$$

Ex. 3.

$$\begin{array}{r} 37) 4.19667(11.347 \\ \underline{37} \\ 49 \\ \underline{37} \\ 126 \\ \underline{111} \\ 156 \\ \underline{148} \\ 82 \\ \underline{74} \\ *8 \end{array}$$

Ex. 4.

$$\begin{array}{r} 12.5) 78.83(----- 6.306 \\ \underline{750} \\ 383 \\ \underline{375} \\ 833 \\ \underline{750} \\ *83 \end{array}$$

$$\begin{array}{r} 12.5 \\ \underline{12.5} \\ 31533 \\ \underline{126133} \\ 630666 \\ \underline{78.8333} \\ \text{proof.} \end{array}$$

Ex.

Ex. 5.

[illegible]

Ex. 6.

$$\begin{array}{r} 8|00)75.4(.09430\bar{3} \\ \underline{72} 800 \\ 34 75.444444 \\ \underline{32} \text{proof.} \\ 24 \\ \underline{24} \\ 44 \\ \underline{40} \\ *4 \end{array}$$

From Ex. 2. and 4. we may learn, that the quot does not always begin to repeat when the repeating figure of the dividend is taken down. And from Ex. 5. it appears, that the quot sometimes comes out a circulate.

In Ex. 5. and 6. the ciphers in the divisor on the right hand are cut off; but the place of the first quotient-figure is determined by that figure of the dividend which would have stood over the units of the first product had the ciphers been retained.

RULE III. If a finite divisor divide a circulating dividend, work as in integers; but in continuing the division, instead of annexing ciphers to the remainder, annex the circulating figures of the dividend.

Ex.

EXAMPLE I.

$$7 \overline{) 3.370.(-481.481}$$

28..

57

56

10

7

*33

28

57

56

10

7

*3

EXAMPLE II.

$$.5 \overline{) 3.7,592.(7.518.5}$$

3.5...5

*25 3.7,592, proof.

25

9

5

42

40

*25

EXAMPLE III.

$$6.5 \overline{) 25.0,714285,(3.857142,8}$$

19.5.....

*557

520

371

325

464

455

92

65

278

260

185

130

*557

EX-

EXAMPLE IV.

7.42)341.50,09756,(46.02439,02
29.68.....

$$\begin{array}{r}
 4470 \\
 4452 \\
 \hline
 *1809 \\
 1484 \\
 \hline
 3257 \\
 2968 \\
 \hline
 2895 \\
 2226 \\
 \hline
 6696 \\
 6678 \\
 \hline
 *1809
 \end{array}$$

RULE IV. If the divisor be interminate, reduce it to a vulgar fraction, as taught in reduction of decimals, Prob. V.; then multiply the given dividend by the denominator, and divide the product by the numerator.

Here there are six cases; for the divisor may either repeat or circulate, and may divide a finite, a repeating, or circulating dividend.

CASE I. When a repeating divisor divides a finite dividend.

EXAMPLE I.

Divide 23.5 by .4 = $\frac{235}{4}$

$$\begin{array}{r}
 9 \\
 \hline
 4 \overline{) 211.5(52.875} \\
 \underline{20} \\
 11 \\
 \underline{8} \\
 35 \\
 \underline{32} \\
 30 \\
 \underline{28} \\
 20 \\
 \underline{20} \\
 0
 \end{array}$$

EXAMPLE II.

Divide 759.011562 by 2.37 = $\frac{214}{90}$

$$\begin{array}{r}
 90 \\
 \hline
 214 \overline{) 68311.040580(319.21047} \\
 \underline{642} \\
 411
 \end{array}$$

The numerator
of the vulgar frac-
tion is found by
subtracting the fi-
nite part, as fol-
lows.

$$\begin{array}{r}
 2.37 \\
 \underline{23} \\
 214
 \end{array}$$

$$\begin{array}{r}
 411 \\
 \underline{214} \\
 1971 \\
 \underline{1926} \\
 450
 \end{array}$$

$$\begin{array}{r}
 450 \\
 \underline{428} \\
 224 \\
 \underline{214} \\
 1005 \\
 \underline{856} \\
 1498 \\
 \underline{1498} \\
 0
 \end{array}$$

The

The product of the dividend into 90, $= 9 \times 10$, may be readily found by the method of multiplying by 9, 99, 999, &c. taught in multiplication of integers, viz multiply first by 10; that is, Move the decimal point one place toward the right hand; from this product subtract the given dividend, and the remainder will be the product of the dividend into 9. Again, multiply this product by 10, by moving the decimal point one place more to the right, and you have the product of the dividend into 90 complete; as in the margin.

$$\begin{array}{r} 7590.11562 \\ 759.011562 \\ \hline 68311.04058 \end{array}$$

The operation, when conducted in this manner, may be made to stand as follows.

$$\begin{array}{r} 2.37 \overline{) 7590.11562} \\ \underline{23} \quad 759.011562 \\ 214 \overline{) 68311.04058} \end{array}$$

EXAMPLE III.

Divide 758.816 by $6.8z = \frac{615}{90}$.

U 2

6.8z

$$\begin{array}{r}
 6.8\cancel{3})7588.16 \\
 \underline{68} \quad 758.816 \\
 615)68293.44(111.046,24390, \\
 \underline{615} \quad \cdot \cdot \cdot \\
 679 \\
 \underline{615} \\
 643 \\
 \underline{615} \\
 2844 \\
 \underline{2460} \\
 3840 \\
 \underline{3690} \\
 *1500 \\
 \underline{1230} \\
 2700 \\
 \underline{2460} \\
 2400 \\
 \underline{1845} \\
 5550 \\
 \underline{5535} \\
 *150
 \end{array}$$

Note 1. If the divisor be .3 or .6, it will be easier to work by the following directions.

1. If the divisor be .3, multiply the dividend by 3, and the product is the quot.

Ex. 1.

$$\begin{array}{r}
 .3)48.75(\\
 \underline{3} \\
 146.25 \text{ quot.}
 \end{array}$$

Ex. 2.

$$\begin{array}{r}
 .03)6.35(\\
 \underline{3} \\
 190.5 \text{ quot.}
 \end{array}$$

2. If the divisor is .6, multiply the dividend by 3, and

and divide the product by 2. Or, multiply the dividend by 1.5 = $\frac{3}{2}$

Ex.

$$\begin{array}{r} .6)82.5(\\ \underline{3} \\ 2)247.5(\\ \underline{123.75} \text{ quot.} \end{array}$$

Or thus,
$$\begin{array}{r} .6)82.5(\\ \underline{1.5} \\ 4125 \\ \underline{825} \\ 123.75 \text{ quot.} \end{array}$$

Note 2. In dividing by a repeating divisor, we may use the contracted form, and have a just quot, provided we extend the first product one place beyond the right of the dividend, and, in subtracting and multiplying, borrow and carry at 9 on the right hand.

Ex. 1.

$$\begin{array}{r} .3)48.75(146.25 \\ \underline{33333} \\ 15416 \\ \underline{13333} \\ 2083 \\ \underline{2000} \\ 83 \\ \underline{66} \\ 16 \\ \underline{16} \\ (0) \end{array}$$

Ex. 2.

$$\begin{array}{r} .6)82.5(123.75 \\ \underline{6666} \\ 1583 \\ \underline{1333} \\ 250 \\ \underline{200} \\ 50 \\ \underline{46} \\ 33 \\ \underline{33} \\ (0) \end{array}$$

The like manner of operation may be used in Cases 2. and 3. following; but the method prescribed in the rule, or in Note 1. is more simple, and much easier.

Note 3. When the divisor is any repeating digit, instead of working by the rule, you may divide 9 by the repeating

repeating digit, and then multiply the dividend by the quot. Ex. 1. done in this manner, follows.

$$\begin{array}{r}
 .4)23.5(\\
 \underline{4)9} \quad (\\
 2.25 \text{ multiplier.} \\
 \underline{1175} \\
 470 \\
 \underline{470}
 \end{array}$$

Quot 52.875 the same as before.

CASE II. When a repeating divisor divides a repeating dividend.

EXAMPLE I.

Divide $43.2\bar{6}$ by $.3 = \frac{3}{10}$

$$\begin{array}{r}
 43.2\bar{6} \\
 \underline{9} \\
 3)389.40(129.8 \\
 \underline{300} \quad \text{quot.} \\
 89 \\
 \underline{60} \\
 29 \\
 \underline{27} \\
 24 \\
 \underline{24}
 \end{array}$$

Or rather thus :

$$\begin{array}{r}
 432.\bar{66} \\
 \underline{43.2\bar{6}} \\
 3)389.40(129.8
 \end{array}$$

EX-

EXAMPLE XII.

Divide 56.034757 by 4.7 = 38

$$4.7 \overline{) 56.034757}$$

$$\underline{4 \quad 56.034757}$$

$$38 \overline{) 504.312820} \quad (13.27139$$

$$\underline{38 \quad 504.312820}$$

$$\underline{124 \quad 504.312820}$$

$$\underline{114 \quad 504.312820}$$

$$\underline{103 \quad 504.312820}$$

$$\underline{76 \quad 504.312820}$$

$$\underline{271 \quad 504.312820}$$

$$\underline{266 \quad 504.312820}$$

$$\underline{52 \quad 504.312820}$$

$$\underline{38 \quad 504.312820}$$

$$\underline{148 \quad 504.312820}$$

$$\underline{114 \quad 504.312820}$$

$$\underline{342 \quad 504.312820}$$

$$\underline{342 \quad 504.312820}$$

EX:

EXAMPLE III

Divide 25293.86 by 45.8 = 410

$$45.8 \overline{) 252938.66}$$

$$45 \overline{) 25293.86}$$

$$410 \overline{) 227644.80} \quad (555.23, 12195,$$

$$205 \dots \dots$$

$$226 \quad 421$$

$$205 \quad 411$$

$$214 \quad 401$$

$$205 \quad 391$$

$$94 \quad 381$$

$$82 \quad 371$$

$$128 \quad 361$$

$$123 \quad 351$$

$$31 \quad 341$$

$$41 \quad 331$$

$$90 \quad 321$$

$$82 \quad 311$$

$$80 \quad 301$$

$$41 \quad 291$$

$$390 \quad 281$$

$$369 \quad 271$$

$$210 \quad 261$$

$$205 \quad 251$$

$$*5$$

EX.

EXAMPLE IV.

Divide 21604.3 by 7.83 = $\frac{705}{90}$

$$7.83 \overline{) 216043.3}$$

$$78 \overline{) 21604.33}$$

$$705 \overline{) 1944390.0} (2758$$

$$1410 \dots$$

$$5343$$

$$4935$$

$$4089$$

$$3525$$

$$5640$$

$$5640$$

Note. When the divisor is .3 or .6, it will be easier to work by the directions subjoined to Note 1. in Case 1.

1. When the divisor is .3, or .03

Ex. 1.

Ex. 2.

$$.3 \overline{) 7583}$$

$$.03 \overline{) 936.86}$$

$$3$$

$$3$$

$$2.2750 \text{ quot.}$$

$$28106.0 \text{ quot.}$$

2. When the divisor is .6, or .06

Ex. 1.

Ex. 2.

$$.6 \overline{) 1928.3}$$

$$.06 \overline{) 85.436}$$

$$3$$

$$3$$

$$2 \overline{) 5785.0}$$

$$2 \overline{) 2563.0}$$

$$2892.5 \text{ quot.}$$

$$1281.5 \text{ quot.}$$

CASE III. When a repeating divisor divides a circulate.

EXAMPLE I.

Divide 92.518, by $.4 = \frac{4}{10}$

$$\begin{array}{r} 92.518, \\ \underline{9} \\ 832.666 \end{array}$$

Or rather thus :

$$\begin{array}{r} 925.185, \\ \underline{92.518,} \\ 4)832.666(208.16 \\ 8 \dots \dots \\ \hline 32 \\ \underline{32} \\ 6 \\ \underline{4} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

EXAMPLE II.

Divide 363.5740, by $48.3 = \frac{483}{10}$

$$\begin{array}{r} 48.3)3635.7407, \\ \underline{48} \quad 363.5740, \\ 435)3272.1666(7.5 \\ \underline{3045} \dots \\ 2271 \\ \underline{2175} \\ 966 \\ \underline{870} \\ 96 \end{array}$$

EX.

EXAMPLE III.

Divide 93.0,07317, by 2.47 = $\frac{223}{90}$

$$\begin{array}{r}
 2.471930.0,73170, \\
 \underline{24 \quad 93.0,07317,} \\
 223)8370.65853,(37.53658, \\
 \underline{669 \quad \cdot \cdot \cdot \cdot \cdot} \\
 1680 \\
 \underline{1561} \\
 *1196 \\
 \underline{1115} \\
 815 \\
 \underline{669} \\
 1468 \\
 \underline{1338} \\
 1305 \\
 \underline{1115} \\
 1903 \\
 \underline{1784} \\
 *119
 \end{array}$$

Note. When the divisor is .3, or .6, it will be easier to work by the directions of Note 1. in Case 1.

Ex. I.

$$\begin{array}{r} .3)7.814, (\\ \underline{3} \\ 23.444 \text{ quot.} \end{array}$$

Ex. 2.

$$\begin{array}{r} .\cancel{0}4.592, (\\ \underline{3} \\ 2)13,\cancel{0}77(\\ \underline{6.888} \text{ quot.} \end{array}$$

CASE IV. When a circulate divides a finite dividend.

X 2

EX-

EXAMPLE I.

Divide 9 by .45, $= 19\frac{8}{9}$

In order to multiply the dividend 9 by 99, I first multiply it by 100, which is done by annexing two ciphers; and from this product I subtract the dividend

$$\begin{array}{r}
 900 \\
 \underline{9} \\
 45) 891 \quad (19.8 \\
 \underline{45} \\
 441 \\
 \underline{405} \\
 360 \\
 \underline{360}
 \end{array}$$

EXAMPLE II.

Divide 81.964 by 4.36, $= 18.78341\phi$

$$\begin{array}{r}
 4.36) 81.964 \\
 \underline{4} \underline{81.964} \\
 432) 8114.436 \quad (18.78341\phi \\
 \underline{432} \\
 3794 \\
 \underline{3456} \\
 3384 \\
 \underline{3024} \\
 3603 \\
 \underline{3456} \\
 1476 \\
 \underline{1296} \\
 1800 \\
 \underline{1728} \\
 720 \\
 \underline{432} \\
 2880 \\
 \underline{2592} \\
 *288
 \end{array}$$

EX.

EXAMPLE III.

Divide 72 by .0,148, $= \frac{148}{9990}$

Here the denominator of the divisor being 9990, I multiply the dividend by 999, and that product by 10.

$$\begin{array}{r}
 72000 \\
 \underline{72} \\
 148)719280(4860 \\
 \underline{592} \\
 1272 \\
 \underline{1184} \\
 888 \\
 \underline{888} \\
 0
 \end{array}$$

CASE V. When a circulate divides a repeating dividend.

EXAMPLE I.

Divide 5.83 by .72, $= \frac{72}{583}$

In order to multiply the dividend by 99, I move the decimal point two places to the right, and then subtract the given dividend.

$$\begin{array}{r}
 583.33 \\
 \underline{5.83} \\
 72)577.50(8.02083 \\
 \underline{576} \\
 150 \\
 \underline{144} \\
 600 \\
 \underline{576} \\
 240 \\
 \underline{216} \\
 *24
 \end{array}$$

EXAMPLE II.

Divide 25.383 by 19.39, $= \frac{1939}{25383}$

$$\begin{array}{r}
 19.39.)2538.866 \\
 \underline{19} \quad 25.388 \\
 1920)2513.280(1.309 \\
 \underline{192} \\
 593 \\
 \underline{576} \\
 1728 \\
 \underline{1728}
 \end{array}$$

EXAMPLE III.

Divide 25293.88 by 555.23, 12195, = $\frac{5552256672}{9999900}$

$$\begin{array}{r}
 555.23, 12195,)252938666.88 \\
 \underline{55523} \quad 25293.88 \\
 5552256672)252936137280(45.8 \\
 \underline{22209026688} \\
 30845870400 \\
 \underline{27761283360} \\
 30845870400 \\
 \underline{27761283360} \\
 *3084587040
 \end{array}$$

CASE VI. When a circulate divides a circulate.

EXAMPLE I.

Divide .962, by .18, = $\frac{18}{99}$

$$\begin{array}{r}
 96.296, \\
 .962, \\
 \underline{} \\
 18)95.833(5.296, \\
 \underline{90} \dots \\
 *53 \\
 \underline{36} \\
 173 \\
 \underline{162} \\
 113 \\
 \underline{108} \\
 *5
 \end{array}$$

EX-

EXAMPLE II.

Divide 534.126, by 4.36, = $\frac{432}{99}$

$$\begin{array}{r}
 4.36 \overline{) 534.12612,} \\
 \underline{4 534.126,} \\
 432 52878.486, (122.4,039,0 \\
 \underline{432 \dots} \\
 967 \\
 \underline{864} \\
 1038 \\
 \underline{864} \\
 1744 \\
 \underline{1728} \\
 *1686 \\
 \underline{1296} \\
 3904 \\
 \underline{3888} \\
 *16
 \end{array}$$

To the remainders I annex
the circulating figures of the
dividend.

EXAMPLE III.

Divide 340.90, by 53.571428, = $\frac{53571375}{999999}$

$$\begin{array}{r}
 53.571428 \overline{) 340909090.90,} \\
 \underline{53 340.90,} \\
 53571375 340908750.00 (6.36, \\
 \underline{321428250} \\
 *194805000 \\
 \underline{160714125} \\
 340908750 \\
 \underline{321428250} \\
 *19480500
 \end{array}$$

When the circle of the quot is likely to run on to many
places, you may stop the operation, and complete the
quot by a vulgar fraction; as in the following example.

EX-

EXAMPLE IV.

Divide 34.56097, by 3.592, = $\frac{3589}{9999}$

3.592,) 34.560.97560,

3 34.56097,

3589) 34526.41463, (9.6200653 $\frac{2724 \overline{46341}}{99999}$

32301

22254

21534

7201 Or, 9.6200653 $\frac{2724 \overline{46341}}{99999}$

7178

23463

21534

The quot would run on to 49 figures of a finite part, and then a circle of 95 places; but I limit it at seven places of decimals, and then complete it by a vulgar fraction; as follows, viz.

19294

17945

13491

10767

(2724)

I complete the partial remainder 2724 by annexing to it the circle of the dividend, and placing both, by way of numerator, over the divisor 3589.

The numerator of this complex fraction being a mixt number, I reduce it to an improper fraction, by multiplying 2724 by the denominator 99999, and adding the numerator 46341 to the product; as in the margin: and then, instead of the mixt number, the numerator of the complex fraction will be $\frac{272443617}{99999}$. Or rather work thus: Esteem 2724.46341, a circulate; and then you find the numerator of the vulgar fraction by subtracting the finite part.

272400000

2724

272397276

46341

272443617

I next divide this fractional numerator by the denominator; which is done by multiplying 3589 by 99999, as in the margin; and now the simple vulgar fraction to be annexed to the partial quot is

$$\begin{array}{r} 358900000 \\ 3589 \overline{) 358896411} \end{array}$$

If the quot thus completed be multiplied by the divisor, it will produce the dividend.

PRACTICAL QUESTIONS.

1. If L. 86, 11 s. be equally divided among 6 men, what is each man's share? *Ans.* L. 14:8:6.

2. Divide L. 684:12:8 among 5 men, so that 4 men may have equal shares, and the 5th only half a share. *Ans.* The divisor is 4.5; the dividend is 684.68; and 4.5)684.68(152.1,407,4 = L. 152:2:9:3 $\frac{1}{5}$ to four men each, and L. 76:1:4:3 $\frac{4}{5}$ to the fifth man.

3. Divide L. 486:16:8 among 4 men, so as 3 men may have equal shares, and the 4th only $\frac{3}{4}$ of a share. *Ans.* The divisor is 3.75; the dividend is 486.88; and 3.75)486.88(129.84.

4. Divide L. 8469, 3 s. among 88 men, so as 87 of them may have equal shares, and one of them only $\frac{1}{3}$ share. *Ans.* The divisor is 87.8, the dividend is 8469.15; and 87.8)8469.15(96.9751.

5. Divide L. 21604:6:8 among 9 men, so as 7 may have equal shares, 1 half a share, and another one third share. *Ans.* $\frac{1}{2} = .5$; and $\frac{1}{3} = .3$; so the divisor is 7.88, the dividend is 21604.8, and 7.88)21604.8(27581.

6. Divide L. 1751, 9 s. among A, B, and C; so as A may have $\frac{3}{4}$, B $\frac{3}{8}$, and C $\frac{3}{5}$.

$$\begin{array}{l} \text{Ans. } \frac{3}{4} = .75 \\ \quad \frac{3}{8} = .375 \\ \quad \frac{3}{5} = .6 \end{array}$$

Divisor 1.725 Dividend 1751.45

And $1.725)1751.45(1015.3 \times \left\{ \begin{array}{l} .75 = 761.5 \text{ A's share.} \\ .375 = 380.75 \text{ B's share.} \\ .6 = 609.2 \text{ C's share.} \end{array} \right.$

1751.45 proof.

7. If, in a year, or 365 days 5 hours and 49 minutes, the sun complete a revolution, or 360 degrees, what is his motion *per* day? *Ans.* 59 min. 8 sec.

8. If 72 C. 2 Q. 4 lb. cost L. 157, 4 s. what will 168 C. 8 lb. cost at that rate? *Ans.* L. 364:4:11.

9. If 25 men do a piece of work in 8 weeks 4 days, how many men will do the same work in 3 weeks and 3 days? *Ans.* 62.5 men; that is, 62 men working daily, and half the daily work of another man.

10. In L. 133:1:4 $\frac{1}{2}$ how many dollars, at 37 $\frac{3}{4}$ d. *per* dollar? *Ans.* 846 dollars.

11. In L. 1066:13:4 Flemish, how many pounds Sterling, at 35 $\frac{5}{8}$ s. *per* pound Sterling? *Ans.* 600 l. Sterling.

12. If 8 $\frac{3}{11}$ cost L. 19:14:4, what will 1 cost? *Ans.* L. 2:7:8.

CHAPTER VII.

DECIMAL PRACTICE.

The price of goods or merchandise may be cast up decimally by any of the methods following.

METHOD I.

Find the decimal of the rate, *viz.* the value of one yard, one pound, one piece, &c.; and this decimal of the rate multiplied into the number or quantity of the goods gives the price.

EXAMPLE I.

At 17 s. *per* yard, what cost 936 yards?

The

The decimal of the rate may be
had readily from the tables.

$$\begin{array}{r} 936 \\ .85 \\ \hline 4680 \\ 7488 \\ \hline 795.60 = 795 \text{ L. } 12 \text{ s.} \end{array}$$

EXAMPLE II.

At 3 s. 4 d. what cost 346?

The decimal of the
rate is $.1\frac{2}{3} = \frac{15}{90}$

$$\begin{array}{r} 346 \\ 15 \\ \hline 1730 \\ 346 \\ \hline 90)5190(57.6 = 57 \text{ L. } 13 \text{ s. } 4 \text{ d.} \\ 45 \\ \hline 69 \\ 63 \\ \hline 60 \\ 54 \\ \hline *6 \end{array}$$

EXAMPLE III.

At 6 s. 8 d. what cost 439?

The decimal of the
rate is $.3$

$$\begin{array}{r} 3)439(146.3 = 146 \text{ L. } 6 \text{ s. } 8 \text{ d.} \end{array}$$

EXAMPLE IV.

At 13 s. 4 d. what cost 766?

The decimal of the
rate is $.6\frac{2}{3}$, so I take $\frac{1}{3}$
twice.

$$\begin{array}{r} 3)766(\\ 255.3 \\ \hline 255.3 \\ \hline 510.6 = 510 \text{ L. } 13 \text{ s. } 4 \text{ d.} \end{array}$$

EXAMPLE V.

At L. 4 : 16 : 8, what cost 75?

Here I multiply the rate
by the number of goods,
as being shortest.

$$\begin{array}{r}
 4.83 \\
 75 \\
 \hline
 241\phi \\
 33833 \\
 \hline
 362.50 = 362 \text{ L. } 10 \text{ s.}
 \end{array}$$

EXAMPLE VI.

At L. 3 : 10 : 10, what cost $38\frac{1}{2}$?

$$\begin{array}{r}
 3.541\phi \\
 38.5 \\
 \hline
 177083 \\
 2833333 \\
 10625000 \\
 \hline
 136.3541\phi = 136 \text{ L. } 7 \text{ s. } 1 \text{ d.}
 \end{array}$$

EXAMPLE VII.

At 14s. 8d. what cost $48\frac{2}{3}$?

The decimal of the
rate is $.73 = \frac{66}{90} = \frac{22}{30}$

$$\begin{array}{r}
 48.6 \\
 22 \\
 \hline
 973 \\
 9733 \\
 \hline
 310)1070.6(\\
 35.68 = 35 \text{ L. } 13 \text{ s. } 9\frac{1}{2} \text{ d.}
 \end{array}$$

EXAMPLE VIII.

At L. 2 : 17 : 9, what cost 16 C. 2 Q. 8 lb.?

$$16.571428,$$

16.571428,
5788.2 multiplier inverted.

331428
432571
13257
1159
82

47.849

$\frac{1}{47.85} = 47 \text{ } 17$ L. s.

METHOD II.

When the rate consists of pence and farthings, find how often it is contained in one pound Sterling, divide the given number of goods by this number, or by its component parts, or work by aliquot parts, and the result will be the price sought.

We confine this method to such rates as consist of pence and farthings, because when the rate consists of shillings, pence, and farthings, or of pounds, shillings, pence, &c. it is shorter and easier to work by Method I.

To make the practice ready and easy, it will be proper to have at hand a table of rates and divisors, such as the following one.

Table of Rates and Divisors.

Rates. 0 Farth.		1 Farth.	2 Farth.	3 Farth.
d.	Divif.	Divif.	Divif.	Divif.
0		3,8,40.	8,60.	8,40.
1	6,40.	8,6,4.	4,40.	4,30,—8.
2	4,30.	4,30,+8.	4,30,+4.	80,—12.
3	80.	80,+12.	80,+6.	80,+4.
4	60.	60,+4 of 4.	60,+8.	60,+8,+2 of 8.
5	6,8.	40,—8.	40,—12.	40,—4 of 6.
6	40.	40,+4 of 6.	40,+12.	40,+8.
7	40,+6.	40,+6,+4 of 6.	40,+4.	40,+4,+6 of 4.
8	30.	30,+4 of 8.	30,+2 of 8.	80,x3,—12 of 80.
9	80,x3.	80,x3,+12 of 80.	80,x3,+6 of 80.	80,x3,+4 of 80.
10	8,3.	30,+4,+8 of 4.	20,—8.	40,+2,+2,+6.
11	20,—12.	40,x2,—8 of 40.	40,x2,—12 of 40.	8,3,+5,—8 of 5.

In the above table the pence stand in the left-hand column, and the farthings on the head, and the divisors in the angle of meeting; which are to be understood and read as follows.

3,8,40. Divide the given number of goods by 3, divide the quot by 8, and again divide this last quot by 40.

4,30,—8. Divide the number of goods by 4, divide the quot by 30, and from this last quot subtract one 8th of itself.

80+12. To an 80th add a 12th of that 80th.

4,30,+4. To a 30th of a 4th add a 4th of that 30th.

60,+4 of 4. Divide by 60, divide again the quot by 4, and to the first quot add a 4th of the second quot.

80x3,—12 of 80. Divide by 80, multiply the quot by 3, and from the product subtract a 12th of the first quot.

EXAMPLE I.

At 1 f. per yard, what cost 432 yards?

$$\text{One } 3d = 144$$

$$\text{One } 8th \text{ of that} = 18$$

$$\text{One } 40th \text{ of that} = .45 = 9s.$$

EXAMPLE II.

At 3 f. what cost 728.5?

$$\text{One } 8th = 91.0625 \quad L. \quad s. \quad d.$$

$$\text{One } 40th \text{ of that} = 2.2765625 = 2 \quad 5 \quad 6\frac{1}{4}$$

EXAMPLE III.

At 1 d. 3 f. what cost 8472.3?

$$\text{One } 4th = 2118.083$$

$$\text{A } 30th \text{ of that} = 70.6027777$$

$$\text{Subtract one } 8th = 8.8253477$$

L. s. d.

$$\text{Ans. } 61.7774308 = 61 \quad 15 \quad 6\frac{1}{2}$$

EX-

EXAMPLE IV.

At 2 d. 2 f. what cost 748.25?

One 4th = 187.0625

A 30th of that = 6.23541666

Add one 4th = 1.55885416

L. s. d. f.

Ans. 7.79427083 = 7 15 10 $2\frac{1}{2}$

EXAMPLE V.

At 4 d. 1 f. what cost 1832.6?

One 60th = 30.544444

One 4th of that = 7.6367

To one 60th add }
one 4th of that } = 1.909027

L. s. d. f.

Ans. 32.453477 = 32 9 0 3.52

EXAMPLE VI.

At 7 d. 1 f. what cost 2854.63,?

One 40th = 71.365909,09,

One 6th of that = 11.894318,18,

One 4th of one 6th = 2.973579,54,

L. s. d.

Ans. 86.233806,81, = 86 4 8

EXAMPLE VII.

At 8 d. 3 f. what cost 794.518,?

One 80th = 9.93,148,

Multiply by 3

29.79444

Subtract one 12th of one 80th = .82762 &c. L. s. d.

Ans. 28.96682 = 28 19 4

EX.

EXAMPLE VIII.

At 11 d. 3f. what cost 854.75?

$$\text{One 8th} = \underline{106.84375}$$

$$\text{One 3d of that} = \underline{35.614583}$$

$$\text{Add one 5th of } \left. \begin{array}{l} \text{one 3d} \end{array} \right\} = \underline{7.122916}$$

$$42.7375$$

$$\text{Subtract one 8th } \left. \begin{array}{l} \text{of one 5th} \end{array} \right\} = \underline{.890364583} \quad \text{L. s. d. f.}$$

$$\text{Ans. } 41.847135416 = 41 \text{ } 16 \text{ } 11 \text{ } 1\frac{1}{4}$$

METHOD III.

The third method is by decimal tables of rates suited to the nine digits; such as those composed and published by the Reverend Mr George Brown in 1718, under the title of *Arithmetica Infinita*, and recommended by Dr John Keill professor of astronomy in the university of Oxford.

These tables are still extant, and extend from 1 farthing to 20 s.; a short specimen of which, with their construction, and the manner of using them, I shall here subjoin.

Decimal Table of Rates, 1 l. the integer.

	Rate.	Rate.	Rate.	Rate.
	s. d.	s. d.	s. d.	s. d.
N.	11 5	11 5 $\frac{1}{4}$	11 5 $\frac{1}{2}$	11 5 $\frac{3}{4}$
1	0.57083	0.571875	0.572916	0.5739583
2	1.1416	1.14375	1.14583	1.147916
3	1.7125	1.715625	1.71875	1.721875
4	2.283	2.2875	2.2916	2.29583
5	2.85416	2.859375	2.864583	2.8697916
6	3.425	3.43125	3.4375	3.44375
7	3.99583	4.003125	4.010416	4.0177083
8	4.56	4.575	4.583	4.5916
9	5.1375	5.146875	5.15625	5.165625

In

In the left-hand column stand the nine digits; and on the right of 1 are the decimals of the respective rates on the head. Thus, .57083 is the decimal of 11 s. 5 d. one pound being the integer; and .571875 is the decimal of 11 s. 5 $\frac{1}{4}$ d. &c. Those decimals opposite to 1 being multiplied through the nine digits, make up or compose the rest of the table.

The superior excellency of tables thus constructed is, that we multiply or divide by 10, 100, 1000, &c. by moving the decimal point so many places to the right or left as there are ciphers in the multiplier or divisor.

Hence the price or value of any number of yards, or other things, denoted by a single digit, or by any of its decuples, may be readily found. Thus, the price of 7, 70, 700, 7000, 70000, 700000 yards, at 11 s. 5 d. per yard, is found as follows.

<i>Yards.</i>	<i>L.</i>	<i>L. s. d.</i>
7 =	3.99583 =	3 19 11
70 =	39.9583 =	39 19 2
700 =	399.583 =	399 11 8
7000 =	3995.83 =	3995 16 8
70000 =	39958.3 =	39958 6 8
700000 =	399583.3 =	399583 6 8

Now, every number may be resolved into decuples of the several digits of which it is composed; find therefore the price of each decuple by itself, as already taught, and their sum will be the price of the whole.

EXAMPLE I.

Required the price of 7956 yards, at 11 s. 5 $\frac{3}{4}$ d. per yard.

<i>Yards.</i>	<i>L.</i>	<i>L. s. d.</i>
7000 =	4017.708333	
900 =	516.5625	
50 =	28.697916	
6 =	3.44375	
	4566.412500 =	4566 8 3

EXAMPLE II.

How much money will one spend in a year, or 365 days, at the rate of 11 s. $5\frac{1}{2}$ d. per day?

Days. L.

$$300 = 171.875$$

$$60 = 34.375$$

$$5 = 2.864583 \quad \text{L. s. d.}$$

$$\hline 209.114583 = 209 \quad 2 \quad 3\frac{1}{2}$$

EXAMPLE III.

If a pensioner's daily pay or wages be 11 s. $5\frac{1}{2}$ d. what will that amount to in 154 days?

Days. L.

$$100 = 57.1875$$

$$50 = 28.59375$$

$$4 = 2.2875 \quad \text{L. s. d.}$$

$$\hline 88.06875 = 88 \quad 1 \quad 4\frac{1}{2}$$

Tables of this sort may be framed for a great variety of useful purposes, and are easily constructed.

Thus, suppose a table wanted for showing the daily income of any annuity, or yearly pension; in this case, divide 1 by 365, and the quot is the income of 1 l. annuity for one day; and by multiplying this quot through the nine digits, the table is constructed, as follows.

The

TABLE.

1	.0,02739726,
2	.0,05479452,
3	.0,08219178,
4	.0,10958904,
5	.0,13698630,
6	.0,16438356,
7	.0,19178082,
8	.0,21917808,
9	.0,24657534,

The use of the table will best appear by examples; which take as follows.

EXAMPLE I.

If one has a yearly pension of 375 l. what is his daily income?

L.

$$300 = .8219$$

$$70 = .1917$$

$$5 = .0136$$

$$1.0272 = 1 \text{ } 0 \text{ } 6\frac{1}{2}$$

L. s. d.

EXAMPLE II.

The yearly rent of a gentleman's estate is 968 l. 10 s. what can he afford to spend *per* day?

L.

$$900 = 2.4657$$

$$60 = .1643$$

$$8 = .0219$$

$$0.5 = .0013$$

$$2.6532 = 2 \text{ } 13 \text{ } 0\frac{3}{4}$$

L. s. d.

If the income for any number of days be required, find the income for one day as above; and multiply the decimal answer by the given number of days. Or, multiply the yearly pension by the given number of days, and use the product as the yearly pension. Thus, in Ex. 2. if the gentleman's income for 64 days be demanded, you may either multiply 2.6532 by 64; or multiply 968.5 by 64; and then work for the product as follows.

Z 2

968.5

968.5

64

38740

58110

61984.0

60000 = 164.3835

1000 = 2.7397

900 = 2.4657

80 = .2191

4 = .0109

L. s. d.

169.8189 = 169 16 4½

The decimals in the table being circles of eight figures, I have used them as approximates, by confining the operations to four decimal places; which, in affairs of this kind, is sufficiently accurate.

If the annual interest of any principal sum be considered as the yearly pension, the interest of the same principal for any number of days may be found by the table as taught above.

The interest of any principal sum for a year is easily found, as being always the hundredth part of the product of the principal multiplied by the rate *per cent*.

EXAMPLE I.

Required the interest for 26 days of 685 l. principal, at 5 *per cent*.

L.

685

5

34.25 annual interest.

26

20550

6850

890.50

800 = 2.19178

90 = .24657

0.5 = .00136 L. s. d.

2.43971 = 2 8 9½

EXAMPLE II.

Required the interest for 80 days of 12000 l. at 4¼ *per cent*.

12000

12000

$4\frac{1}{4}$

48

3

510.00 annual interest.

80

40800

40000 = 109.58904

800 = 2.19178 L. s. d.

111.78082 = 111 15 $7\frac{1}{2}$

D U O D E C I M A L S.

Decimal practice may be used with great advantage in the multiplication and division of duodecimals, where the integer is divided into twelve equal parts, called *primes*, and each prime into twelve seconds, each second into twelve thirds, &c.

For the ready conversion of primes, seconds, thirds, &c. into decimals of the integer, the following table is constructed.

Decimal table of primes, seconds, &c.

N.	Primes. '	Seconds. "	Thirds. '''	Fourths. '''
1	.083	.00694	.000578,703,	.00004822
2	.16	.0138	.001157,407,	.00009645
3	.25	.02083	.001736,111,	.00014467
4	.33	.0277	.002314,814,	.00019290
5	.416	.03472	.002893,518,	.00024112
6	.5	.0416	.003472,222,	.00028935
7	.583	.04861	.004050,925,	.00033757
8	.66	.0555	.004629,629,	.00038580
9	.75	.0625	.005208,333,	.00043402
10	.83	.0694	.005787,037,	.00048225
11	.916	.07638	.006365,740,	.00053047

In

In the column of fourths, the decimals run on to eight places of a finite part, and nine figures of a circle; but the finite part by itself, which alone is inserted in the table, will be found sufficient; and in the column of thirds too, the circle of three figures may in most cases be neglected.

I. MULTIPLICATION.

EXAMPLE I.

What is the product of 247 by 18 5

$$\begin{array}{r} 24 \cdot 7 = 24.58\bar{3} \\ 18 \cdot 5 = 18.41\bar{6} = \frac{16575}{900} \end{array}$$

Or thus :

$\begin{array}{r} 24.58\bar{3} \\ 16575 \\ \hline 12291\bar{6} \\ 17208\bar{3}3 \\ 12291\bar{6}66 \\ 147500000 \\ 2458\bar{3}3333 \\ \hline 9 00) 407468.750 \\ \hline 452.7430\bar{8} \\ 12 \\ \hline 8.91\bar{6}66 \\ 12 \\ \hline 11.000 \end{array}$	$\begin{array}{r} 24.58\bar{3}33 \\ 14.81 \\ \hline 2458333 \\ 1966666 \\ 98333 \\ 2458 \\ \hline 452.5790 \\ \frac{1}{3} = 819 \\ \frac{1}{3} = 819 \\ \hline 452.7428 \\ 12 \\ \hline 8.9136 \\ 12 \\ \hline 10.9632 \end{array}$
--	---

Ans. 452 8 11

In

In working by the inverted method, for the repeating ϕ in the multiplier, I take $\frac{2}{3}$ of the multiplicand. The result wants very little of the true answer.

EXAMPLE II.

Multiply 18 6 by 2 4, and 2 3 continually.

$$18\ 6 = 18.5$$

$$2\ 4 = 2.3$$

$$2\ 3 = 2.25$$

$$2.3$$

$$18.5$$

$$11\phi$$

$$18\phi 6$$

$$2333$$

$$43.1\phi$$

$$2.25$$

$$21583$$

$$86333$$

$$863333$$

$$97.1250$$

$$12$$

$$Ans. 97\ 1\ 6$$

$$1.500$$

$$12$$

$$6.0$$

EXAMPLE III.

What is the square of 6 9 4 = 6.7 = $\frac{61}{9}$?

$$6.7$$

$$\begin{array}{r}
 6.7 \\
 61 \\
 \hline
 67 \\
 4086 \\
 \hline
 9)413.4(\\
 45.938271604, \\
 \hline
 45.938271604, \\
 \hline
 11.259,259,259, \\
 \hline
 12 \\
 \hline
 3.111 \\
 \hline
 12 \\
 \hline
 1.2 \\
 \hline
 12 \\
 \hline
 4.0
 \end{array}$$

Ans. 45 11 3 1 4

EXAMPLE IV.

What is the product of 36 9 9 by 4 8?

$$\begin{array}{r}
 36 \ 9 \ 9 = 36.8125 \\
 4 \ 8 = .32 = \frac{32}{100} \\
 \hline
 36.8125 \\
 35 \\
 \hline
 1840625 \\
 1104375 \\
 \hline
 9)1288.4375(\\
 14.315974 \\
 \hline
 12 \\
 \hline
 3.791866 \\
 \hline
 12 \\
 \hline
 9.5000 \\
 \hline
 12 \\
 \hline
 6.0
 \end{array}$$

Ans. 14 3 9 6

II. DIVISION.

EXAMPLE I.

$$\begin{array}{l} \text{Divide } 452 \text{ } 8 \text{ } 11 = 452.74308 \\ \text{by } 18 \text{ } 5 = 18.418 = \frac{16575}{900} \end{array}$$

$$\begin{array}{r} 18.418) 4527.43085 \\ \underline{1841} \quad 452.74308 \\ 16575) 407468.750(24.583 \\ \underline{33150} \quad \dots \quad 12 \\ 75968 \quad 7.000 \\ \underline{66300} \\ 96687 \\ \underline{82875} \\ 138125 \\ \underline{132600} \\ 55250 \\ \underline{49725} \\ *5525 \end{array}$$

Ans. 24 7

EXAMPLE II.

$$\begin{array}{l} \text{Divide } 97 \text{ } 1 \text{ } 6 = 97.125 \\ \text{by } 2 \text{ } 3 = 2.25 \text{ and the quot} \\ \text{by } 18 \text{ } 6 = 18.5 \end{array}$$

$$\begin{array}{r}
 18.5) \\
 2.25) 97.125 (43.1\bar{6} (2.3 \\
 \underline{900} \quad \underline{370} \quad \underline{12} \\
 712 \quad 616 \quad 4.0 \\
 \underline{675} \quad \underline{555} \\
 375 \quad *61 \\
 \underline{225} \\
 1500 \\
 \underline{1350} \quad \text{Ans. } 2 \quad 4 \\
 *150
 \end{array}$$

EXAMPLE III.

$$\begin{array}{l}
 \text{Divide } 14 \quad 3 \quad 9 \quad 6 = 14.31597\bar{7} \\
 \text{by } 36 \quad 9 \quad 9 \quad = 36.8125
 \end{array}$$

$$\begin{array}{r}
 36.8125) 14.31597\bar{7} \quad (.3\bar{8} \\
 \underline{1104375} \quad \underline{12} \\
 3272222 \quad 4.66 \\
 \underline{2945000} \quad \underline{12} \\
 *327222 \quad 8.0 \quad \text{Ans. } 4 \quad 8
 \end{array}$$

EXAMPLE IV.

$$\begin{array}{l}
 \text{Divide } 65 \quad 9 \quad 11 \quad 0 = 65.8263\bar{8} \\
 \text{by } 8 \quad 1 \quad 10 \quad 11 = 8.159142
 \end{array}$$

8.159142)

$$8.159142)65.826388(8.067808$$

$$\begin{array}{r} \dots\dots 65273136 \\ \hline \end{array}$$

$$553252$$

$$489548$$

$$63704$$

$$57113$$

$$\text{Ans. } 8 \text{ } 0 \text{ } 9 \text{ } 8 \text{ } 2$$

$$6591$$

$$6527$$

$$64$$

$$64$$

EXAMPLE V.

$$\begin{array}{l} \text{Divide } 15 \text{ } 0 \text{ } 0 \text{ } 0 = 15.000000 \\ \text{by } 3 \text{ } 6 \text{ } 11 \text{ } 9 = 3.581596 \end{array}$$

$$3.581596)15.000000(4.188079$$

$$\begin{array}{r} \dots\dots 14326384 \\ \hline \end{array}$$

$$673616$$

$$358159$$

$$315457$$

$$286527$$

$$28930$$

$$28652$$

$$278$$

$$250$$

$$28$$

$$\text{Ans. } 4 \text{ } 2 \text{ } 3 \text{ } 1$$

$$27$$

$$1$$

PRACTICAL QUESTIONS.

1. In a log of timber, whose length is 6 feet 3 inches, breadth 3 feet 8 inches, and thickness 2 feet 3 inches, how many cubic feet, and what the value, at

A a 2

6 s. 9 d.

6 s. 9 d. per cubic foot? *Ans.* 51 cubic feet 6' primes 9 seconds, and its value L. 17 : 8 : 0 : 2 $\frac{1}{4}$.

2. The area of a rectangular floor is 919 square feet 8 primes 11 seconds, and the length of it is 38 feet 9 inches 6 lines; what is the breadth? *Ans.* 23 feet 8 inches 6 lines.

3. The solid content of a parallelopiped is 86 cubic feet 7 primes 9 seconds 6 thirds, its length is 17 feet 6 inches, and its breadth 1 foot 11 inches; what is the thickness? *Ans.* 2 feet 7 inches.

SEXAGESIMALS.

Decimal practice might likewise be used to good purpose in the arithmetic of sexagesimals, as it would shorten and facilitate the operations.

Sexagesimals, strictly speaking, are degrees, minutes, seconds, thirds, &c. where each degree is divided into 60 minutes, and each minute into 60 seconds, &c.; but under this title is also usually comprehended the division of a sign into 30 degrees. They are commonly marked as under.

Signs. deg. min. sec. thirds.

° ' " "

7 24 36 54 48 &c.

Sexagesimals properly belong to astronomy, being used in computations of motion and time, where the degree of motion, and hour of time, are equally divided into 60 minutes. The preference of the decimal method to that of the sexagesimal will appear from the following example of addition done both ways.

<i>Sexagesimally.</i>	<i>Decimally.</i>
<i>Signs. ° ' " S.</i>	
10 20 47 17 =	10.69293,518,
7 18 50 40 =	7.62814,814,
9 25 30 20 =	9.85018,518,
11 10 40 50 =	11.35601,851,
3 15 49 7 =	3.52728,703,

From

From the above example it is obvious, that even in addition the decimal operation is more simple and easy than the sexagesimal, especially if care be taken to use no more decimal places than what are absolutely necessary.

But in multiplication and division the advantage of the decimal method is still greater; for in the sexagesimal way the operation is extremely tedious; whereas, by working decimally, it is performed in the same manner, and with the same ease, as in duodecimals.

VULGAR FRACTIONS.

Decimal practice may sometimes be profitably used in the arithmetic of vulgar fractions, the operation being shorter and easier in the decimal than in the vulgar way. This I shall illustrate by a few examples.

I. A D D I T I O N.

Ex. 1. What is the sum of $\frac{1}{4} + \frac{2}{3}$ l.?

$$\begin{array}{r} \frac{1}{4} = .25 \\ \frac{2}{3} = .666 \\ \hline .916 = 18 \text{ s. } 4 \text{ d.} \end{array}$$

Ex. 2. What is the sum of $14\frac{7}{8} + 18\frac{2}{3} + \frac{3}{4}$ of $\frac{5}{8}$ C.?

$$\begin{array}{r} \text{C.} \\ 14\frac{7}{8} = 14.875 \\ 18\frac{2}{3} = 18.6666 \\ \frac{3}{4} \text{ of } \frac{5}{8} = \frac{15}{32} = .46875 \\ \hline 34.1066 = 34 \text{ C. } 0 \text{ Q. } 18\frac{2}{3} \text{ lb.} \end{array}$$

Ex. 3. What is the sum of $35\frac{1}{2} + 24\frac{1}{3} + 12\frac{3}{11}$ lb. Troy?

$$\begin{array}{r} \text{lb.} \\ 35\frac{1}{2} = 35.5 \\ 24\frac{1}{3} = 24.333 \\ 12\frac{3}{11} = 12.2727 \\ \hline 72.106 = 72 \text{ lb. } 1 \text{ oz. } 5\frac{5}{11} \text{ dw.} \end{array}$$

II. SUB.

II. SUBTRACTION.

Ex. 1. From $\frac{3}{4}$ subtract $\frac{1}{8}$ l.

L.

$$\frac{3}{4} = .75$$

$$\frac{1}{8} = .125$$

$$\begin{array}{r} .75 \\ -.125 \\ \hline .625 \end{array} \quad \begin{array}{c} s. \\ d. \end{array}$$

$$.41\bar{6} = 8 \quad 4$$

Ex. 2. From $\frac{2}{3}$ of $\frac{7}{8}$ subtract $\frac{1}{2}$ of $\frac{1}{4}$ lb. Troy.

lb.

$$\frac{2}{3} \text{ of } \frac{7}{8} = \frac{7}{12} = .58\bar{3}$$

$$\frac{1}{2} \text{ of } \frac{1}{4} = \frac{1}{8} = .125 \quad \text{oz. dw.}$$

$$.458\bar{3} = 5 \quad 10$$

Ex. 3. From $36\frac{1}{3}$ subtract $22\frac{2}{3}$ days.

Days

$$36\frac{1}{3} = 36.3\bar{3}$$

$$22\frac{2}{3} = 22.8\bar{3}$$

D. h.

$$13.50 = 13 \quad 12$$

III. MULTIPLICATION.

Ex. 1. Multiply $19\frac{7}{12}$ by $22\frac{1}{3}$ feet.

F.

$$19\frac{7}{12} \cdot \cdot \cdot = 19.58\bar{3}$$

$$22\frac{1}{3} = 22.\bar{3} = 22\frac{1}{3}$$

201

$$1958\bar{3}$$

$$3916\bar{6}66$$

$$9)3936.250$$

Sq. f. sq. in.

$$437.36\bar{1} = 437 \quad 52$$

Ex. 2. How many solid yards of digging in a cellar that is $8\frac{2}{3}$ yards long, $5\frac{1}{2}$ yards wide, and $2\frac{1}{4}$ yards deep?

Tds.

Rds.

$$8\frac{2}{3} = 8.6$$

$$5\frac{1}{2} = 5.5$$

$$2\frac{1}{4} = 2.25$$

438

4383

47.66

2.25

2388

9533

95333

$$107.250 = 107\frac{1}{4} \text{ solid yards.}$$

Ex. 3. If a cistern be 12 feet long, 7 feet wide, and $3\frac{1}{2}$ feet deep, how many wine-gallons will it hold, the gallons in a solid foot being 7.480519,?

F.

12

7

84

3.5

420

252

294.0

7.480519,

294

29,922,077,

673,246,753,

1496,103,896,

W. gal.

$$2199.272727, = 2199\frac{3}{11}$$

IV. DIVISION.

Ex. 1. Divide $6\frac{7}{18}$ l. by $\frac{4}{11}$.

$$\frac{4}{11} = .36, \text{ and } 6\frac{7}{8} = 6.3\frac{1}{2}$$

$$.36)638.88($$

$$\underline{00} \quad 6.3\frac{1}{2}$$

L. s. d.

$$36)632.50(17.5694 = 17 \ 11 \ 4\frac{2}{3}$$

$$\underline{36} \quad \dots$$

$$272$$

$$\underline{252}$$

$$205$$

$$\underline{180}$$

$$250$$

$$\underline{216}$$

$$340$$

$$\underline{324}$$

$$160$$

$$\underline{144}$$

$$*16$$

Ex. 2. If the solid content of a cellar be $107\frac{1}{4}$ yards, its breadth $5\frac{1}{2}$ yards, and its depth $2\frac{1}{4}$ yards, what is its length?

$$5.5)107.25(19.50(8.6 = 8\frac{2}{3} \quad \text{Yds.}$$

$$\underline{55}$$

$$\underline{1800}$$

$$522$$

$$\underline{1500}$$

$$\underline{495}$$

$$\underline{1350}$$

$$275$$

$$*150$$

$$275$$

Ex. 3. Divide $43\frac{7}{11}$ by $\frac{4}{5}$ of $\frac{2}{3}$.

$$43\frac{7}{11} = 43.63, \text{ and } \frac{4}{5} \text{ of } \frac{2}{3} = \frac{8}{15} = .53$$

.53)

$$\begin{array}{r}
 .58)436.36, \\
 \underline{5 \quad 43.63,} \\
 48)3927.27, (81.81, = 81 \frac{9}{11} \\
 \underline{384 \quad \cdot \cdot} \\
 87 \\
 \underline{48} \\
 *392
 \end{array}$$

RULE OF THREE DIRECT.

Decimal practice is frequently the shortest and easiest method of operation in the rule of three.

EXAMPLE I.

If C. 3 : 1 : 14 of raisins cost L. 10 : 2 : 6, what will 6 C. 3 Q. cost at that rate?

	C.	Q.	lb.	L.	s.	d.	C.	Q.
Vulgar state	3	1	14	: 10	2	6	:: 6	3
Decimal state	3.375			: 10.125			:: 6.75	

$$\begin{array}{r}
 6.75 \\
 \hline
 50625 \\
 70875 \\
 60750 \\
 \hline
 3.375)68.34375 (20.25 = 20 \quad 5 \\
 \underline{6750 \quad \cdot \cdot \cdot} \\
 8437 \\
 \underline{6750} \\
 16875 \\
 \underline{16875}
 \end{array}$$

EXAMPLE II.

If a wedge of gold, weighing 14 lb. 3 oz. 8 dw. cost L. 514, 4s. what is that per ounce?

lb. oz. dw. L. s. oz.

Vulgar state 14 3 8 : 514 4 :: 1

Decimal state 14.283 : 514.2 :: .083

514.2

166

3333

8333

416666

14.283)42.8500(

1428 42.8500 L.

12855)38565.0(3 Ans.

38565

EXAMPLE III.

If 1 yard cost 8 s. 4 d. what will $3482\frac{1}{3}$ cost?

r. s. d. r.

Vulgar state 1 : 8 4 :: 3482 $\frac{1}{3}$

Decimal state 1 : 416 :: 3482.3

41

375

375

174116

2437633

10447000

9|00)1305875.0

L. s. d.

1450.977 = 1450 19 $5\frac{1}{3}$

EXAMPLE IV.

If 15 hogheads 3 firkins and 8 gallons of beer cost
 L. 24 : 13 : 4, what is that per hoghead, and per
 gallon?

Hhd.

	Hhd.	f.	g.	L.	s.	d.	Hhd.
Vulgar state	15	3	8	: 24	13	4	:: 1
Decimal state	15.6,481,	:	24.6666	:	:	:	1
	1561		24.6		54)		
	156325)		246426.6		(1.57637	(.02919	
			156325		108...		
			901016		496		
			781625		486		
			1193916		103		
			1094275		54		
			996416		497		
			937950		486		
			584666		11		
			468975				
			1156916				
			1094275				
			62641				

Ans. L. s. d.
 1.57637 = 1 11 6 $\frac{1}{4}$ per hhd.
 .02919 = 7 per gallon.

RULE OF THREE INVERSE.

EXAMPLE I.

If I borrow L. 64 for 8 months, what sum lent for 12 months, or a year, will requite the favour?

B b 2

r.

r. L. r.

Vulgar state $1 : 64 :: \frac{8}{12}$ Decimal state $1 : 64 :: .6$

6

L. s. d.

9)384(42.6 = 42. 13 4

36

24

18

Or thus :

60

3)64(21.3

54

21.3

6

The same 42.6 as before.

EXAMPLE II.

If 40 poles or rods in length and 4 in breadth make an acre, what must be the length to make an acre when the breadth is 5 poles 1 yard?

P. r. P. P.

Vulgar state $5 \ 1 : 40 :: 4$ Decimal state $5.18, : 40 :: 4$

4

5.18,)160.00(

5

160

P. y. f. in.

513)15840(30.877 = 30 4 2 5

1539

4500

4104

3960

3591

3690

3591

99

COM.

D. M. A. M. D.
 Vulgar state $\frac{2}{3} \times 2 : \frac{3}{4} :: 6 \times 3\frac{1}{3}$

Decimal state $.6 \times 2 : .75 :: 6 \times 3.3$

$\frac{2}{12} : \frac{20}{15.00} :: \frac{6}{20.0}$
 12) 15.00 (20.0
 1 15.
 ———
 12) 135.0 (11.25 = 11 1
 12
 ———
 15
 12
 ———
 30
 24
 ———
 60
 60

Acres. Rood.

EXAMPLE III.

If 120 men in 20 days drink 42 hogheads 3 firkins and 4 gallons of beer, how much will 80 men drink in 50 days at that rate?

D. M. H. f. g. M. D.

V. state $20 \times 120 : 42 \quad 3 \quad 4 :: 80 \times 50$

D. state $20 \times 120 : 42.5, 740, :: 80 \times 50$

$\frac{20}{2400} : \frac{4000}{170296.296} :: \frac{80}{70.95679} = 70 \quad 5 \quad 6 \quad 6$
H. f. g. p.

229

216

136

120

162

144

189

168

216

216

EX.

EXAMPLE IV.

If 4 men cast a ditch $8\frac{1}{2}$ feet long, $3\frac{1}{3}$ deep, and $2\frac{1}{4}$ broad, in $9\frac{1}{3}$ days; in how many days will 8 men cast a ditch $25\frac{1}{2}$ feet long, $6\frac{2}{3}$ deep, and $4\frac{1}{2}$ broad?

Vulgar state $b. d. l. \overset{\cdot}{M}. \overset{\cdot}{D}. \overset{\cdot}{M}. l. d. b.$
 $2\frac{1}{4} \times 3\frac{1}{3} \times 8\frac{1}{2} \times 8 : 9\frac{1}{3} :: 4 \times 25\frac{1}{2} \times 6\frac{2}{3} \times 4\frac{1}{2}$

Dec. state $2.25 \times 3.3 \times 8.5 \times 8 : 9.3 :: 4 \times 25.5 \times 6.6 \times 4.5$

2.25	3060	4.5
168	560	338
666	280	2666
6666	28560	30.00
7.500		25.5
8.5		765.0
375		4
600		3060
63.75		
8		

510.00) 28560 (56 days. *Ans.*

2550

3060

3060

The End of the DECIMAL ARITHMETIC.

Class VI. PROBABLY IN AOTIC.

۷۱

51

1

35

10

10

10

which is 1 foot long, 6 inches deep, and 1 inch broad; in 1/2 days; in how many days will 8 men can a ditch 1 1/2 foot long, 6 inches deep, and 1 inch broad?



0000

1

10

10

10

—

100

1998

2

2

•

10

1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 26

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